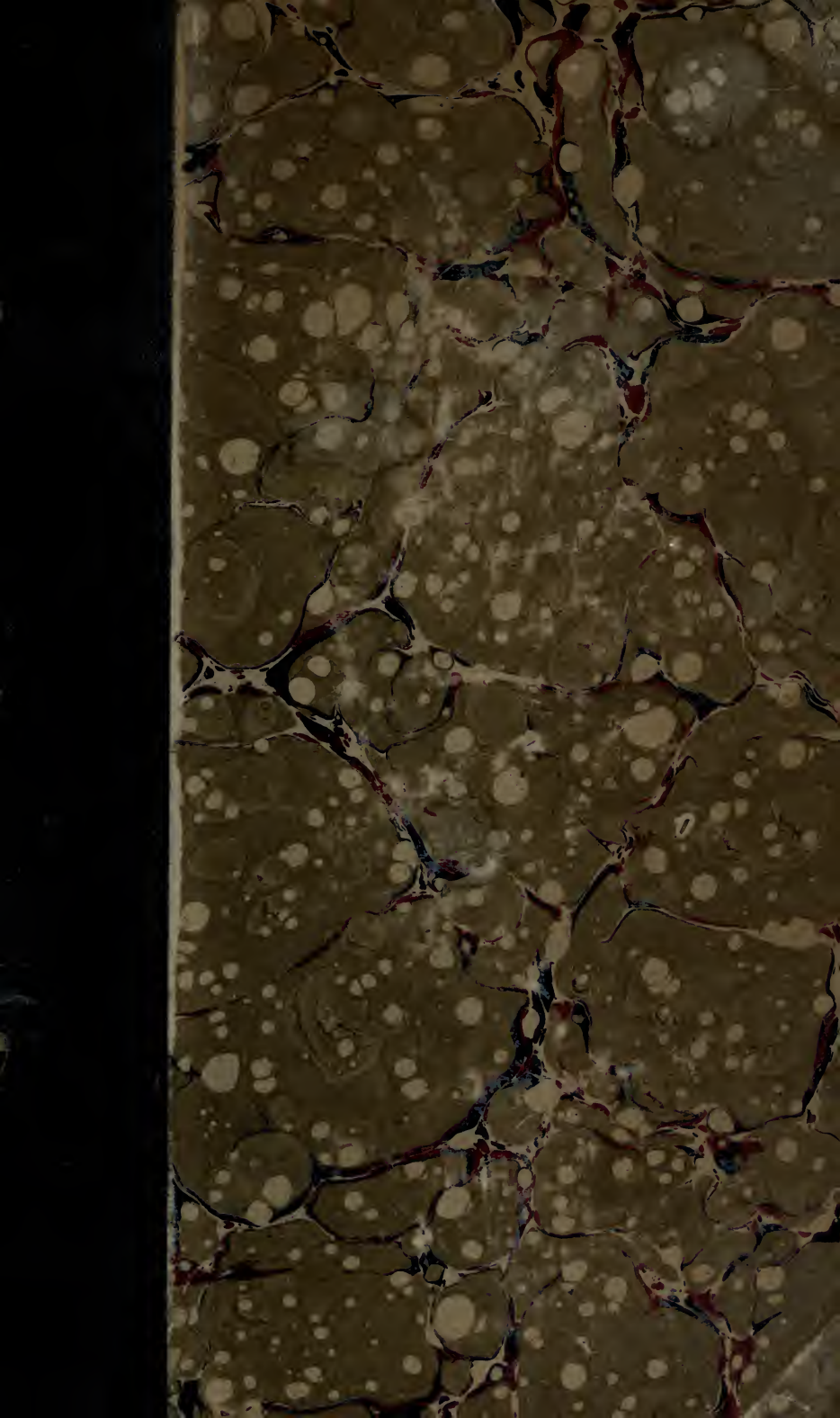






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MATHEMATICS SIMPLIFIED,

AND

PRACTICALLY ILLUSTRATED,

BY

THE ADAPTATION OF THE PRINCIPAL PROBLEMS

TO

THE ORDINARY PURPOSES OF LIFE,

AND BY

A PROGRESSIVE ARRANGEMENT,

APPLIED TO THE MOST FAMILIAR OBJECTS,

IN THE PLAINEST TERMS:

TOGETHER WITH

A COMPLETE ESSAY

ON

THE ART OF SURVEYING LANDS, &c.

BY

SUCH SIMPLE INVENTIONS,

AS MAY FOR EVER BANISH THE NECESSITY OF COSTLY

AND COMPLEX INSTRUMENTS.

By CAPT. THOMAS WILLIAMSON,

AUTHOR OF THE WILD SPORTS OF INDIA.

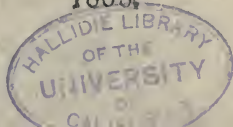
“ While the Continent has supplied us with most elaborate and useful Treatises on various Articles in Physic, Astronomy, Practical Mechanics, Hydraulics, and Optics, there have not appeared in Britain half a Dozen Treatises worth consulting, for the last Forty Years.”

Dr. Robinson.

LONDON: .

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MATHEMATICS SIMPLIFIED.

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A COMPLETE ESSAY

THE ART OF FISHING LANDS

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P R E F A C E.

NOTWITHSTANDING the perfection to which the majority of the arts and sciences have been carried, yet it is acknowledged by all considerate persons, that British handicrafts are by no means so well informed, as to the principles of their respective operations, as the excellent workmanship they produce would lead us to suppose! The fact is, that the mechanic is a mere automaton, under the guidance of a skilful director, in whose absence, therefore, either the work must stop, or there must be a perpetual risk of error, and of consequent loss.

If we except such as have served regular apprenticeships under men of superior ability, and those who have laboured in great cities, or at extensive manufactories, where knowledge *will* flow in upon the mind, few, indeed, are the artizans that can account for any one proportion, or form, they habitually construct: the smallest deviation occasions hesitation at least, if it does not completely derange the operator; and such is the obstinacy of ignorant men, that, even when they do follow the directions they receive from their employers, it is, usually, with a bad grace, if not with a bad will; such as rarely fails to injure, and perhaps to frustrate the good effects of the proposed improvement.

Admitting, that not only a good disposition, but a quick intelligence should exist, still, without some knowledge of

the mathematics, the workman will either be unable to execute the job, or he will proceed by such circuitous means, as must expend more materials, as well as more time, and, after all, without arriving so exactly at the desired point, as one who should have some acquaintance with the principles of formation.

The generality of persons employed in mechanical operations, can read, print, and many can write well enough to be understood; they know the uses of their tools, and can apply them with great accuracy; but their minds require to be enlarged, so as to embrace the whole plan, instead of being confined to the mere nicety of fine shapes, and close joints. Many of the country wrights, &c. possess strong talents, which, being uncultivated, occasion them to take a wrong turn: whence we often find such persons commence speculations that terminate in their ruin, but which, aided by a certain portion of mathematical knowledge, might have raised them both to fortune, and to fame.

In the ordinary way of schooling, especially in small towns, where the teaching is cheap, or gratuitous, we cannot expect to find any learning of this nature; in truth, we do not see many of the superior academies, where useful knowledge, if at all considered, forms such a prominent feature as might be expected in a mercantile country, whose exports amount to such an immense value, and whose productions are wrought so highly. It must be confessed, that, to young persons intent more on pastime than on improvements, mathematics, in the form they have hitherto appeared, ordinarily present no very great inducements to application; on the contrary, they have been, in most cases, considered among the pupils, as that kind of task, which afforded more of the irksome,

than of the recreative; and, when acquired, did not sufficiently display their utility and importance, especially among those who, being *born gentlemen*, look down with contempt on whatever appertains to business or to labour.

I never met with a publication on this subject that had not one of these defects; it was either too general, too bulky, and too expensive, or it was too brief, too pedantic, and too refined, for the purposes of such as possessed but common intellect. Nor can I say that I ever saw a mathematical work, intended for general use, that proceeded in an easy, familiar, and progressive manner, so as to lead the student pleasingly through the problems, whose order always presented a succeeding novelty, of which all the parts were previously understood.

It is not proposed to uphold this volume as presenting what relates to the more abstruse branches of the science; it is offered as a prefatory course, for such as mean to follow up the study to the fountain-head, while it will be found to contain every thing essential to the generality of mechanics, and to afford some few hints as to the formation of ornaments, &c.

The plates are constructed on a plan conformable to the economy of the publication, and, it is hoped, will, by their number, evince a very earnest desire to give every instruction suited to the intent of the work in general.

DIRECTIONS TO THE BINDER.

Let Plate 1	be bound in at Page 12,	{ at the end of the Definitions.
— 2	— 36,	after Problem 15.
— 3	— 42,	— 19.
— 4	— 64,	— 28.
— 5	— 74,	— 36.
— 6	— 94,	— 48.
— 7	— 105,	— 56.
— 8	— 112,	{ at the end of the Axioms.
— 9	— 130,	after Example 6.
— 10	— 135,	before Example 9.
— 11	— 141,	— 10.
— 12	— 146,	after Example 14.
— 13	— 164,	— 22.
— 14	— 152,	— 17.
— 15	— 156,	— 21.
— 16	— 168,	— 27.
— 17	— 170,	— 30.
— 18	— 174,	— 31.
— 19	— 194,	— 33.
— 20	— 204,	{ after Surveying on Emergency, and before Irrigation.
— 21	— 207,	{ after Irrigation, and before Mines.
— 22	— 222,	{ to conclude the work.
— 23,	being supplementary, to face the page ¹ / _{of} Errata.	

CORRECTIONS OF ERRATA,

AND

REFERENCES TO THE SUPPLEMENTARY PLATE.

The Publishers lament that the Plates were inadvertently put to press before the corrections were made; and the great delay and extra expence which would have arisen from waiting for impressions from the corrected plates, have rendered it expedient to add a Supplementary one.

Definition 8. should be lettered as seen in the Plate.

Definition 15. should be lettered, and have the tangent A C, as in the Plate.

Definition 18. wants a C at the left end of the base line.

Definition 16. wants a at the centre, and b c at the end of the horizontal diameter, as also d at the summit of the vertical diameter.

Definition 42. wants a at the summit; b and c at the end of the base line, and d at its centre.

All these are in Plate I.

In Plate II. the second Problem wants its number,

The second Figure, which shews how to ascertain the centre of an inclined Cone, was altogether omitted, but will be found in the Supplementary Plate.

Fig. 1. of the 6th Problem was omitted, but will be found in the Supplementary Plate.

Fig. 4. shewing the scale of 10 parts, should be lettered a, a, a, at the several angles on the base line, and b, b, b, opposite the end of the horizontals, respectively.

In Plate III. the sixteenth Problem wants a line, from b to d, in the centre of which, should be a line, g, proceeding to the centre, f.

In Fig. 2. of the nineteenth Problem, the letter f is wanting, at the summit of the triangle, a, d, f.

In Plate IV. the designation of Fig. 4. is wanting to the 4th Figure of twenty-third Problem.

The Parallelograms 1 and 2 in Fig. 1. of 23, should have stood on equal bases.

In Plate V. the letter e outside the circle, in the thirty-second Problem, should have been omitted.

In Fig. 2. of the 29th Problem, the letter c is wanting, at the top of the crucifix.

In Plate VI. in the thirty-eighth Problem, the letter b is wanting at that angle of the pentagon which is unlettered. In 41, the letter b is also omitted in the same instance, as is f, which should be in the centre of that Figure. In 40, the letter g is wanting at the centre of the upper line.

The triangle appertaining to the forty-eighth Problem, should be marked Fig. 1. and the bisected circle should be Fig. 2.; at the right hand upper corner of the 5th Parallelogram, in the forty-sixth Problem, the letter k is wanting.

ERRATA.

In Plate VII. in the 51st Prob. for letter *a*, read *b*. In 52d, the left end of the base line should be marked *p*, and the right end *e*. In 53th, *k* is wanted at the left end of the base line, and the line *b e* should be continued to *k*. In 54th, the letter *a* should be placed where the line from falls upon the line *p e*.

In Plate VIII. the first Axiom wants the Fig. 1; the 13th wants a line from the left hand upper corner, to the right hand lower corner, and from the same to the farther point of the lower triangle, as in the Supplementary Plate.

In Plate IX. the Standard Triangle should be lettered as shewn in the Supplementary Plate, Fig. 1.

Fig. 2. should have *a* over the loop end of the iron.

Fig. 3. should have *o* above the small perforation.

In Example 2. where the two diameters intersect, there should be the letter *o*, and where the diagonal *k k* is intersected, there should be the letter *p*.

In Fig. 2. of Example 5, at the intersection of the dotted lines, there should be the letter *q*.

In Plate X. Example 8. should have a dotted line from *d* to *c*.

Plate. XI. the circle describing the figure of the plumb weight, should have the letter *a* in its centre.

In Plate XII. at the point of the Standard Triangle, in Example 12, there should be the letter *l* to the left of *q*.

In Plate XIII. at the upper angle of the standard, near the stud, there should be the letter *d*.

In Plate XV. The small house below that at *d*, ought to be lettered *g*; the Fig. 6. should have been at the very point opposite the flag-staff.

In Plate XVIII. In South-Fold, the line measuring 76 is wanting, it should be perpendicular to that of 225, and proceed to the corner of the field, passing close by the number 76. The field in which the cottage stands (at the bottom of the Plate) should be "*Brent's Farm*."

In Plate XIX. The lower Plane Table ought to have the letter *a*; and the lines from that, and the Plane Table *b*, ought to be numbered according to their respective lengths.

In Plate XX. where the four lines fall from the parallel, *f e*, upon the base line *b a*, there should be the letters *c, c, c, c*.

N. B.—The Reader is requested to make as many of the foregoing corrections as may be practicable, in the several Plates, with a pencil, &c. before he studies the Problems to which they refer.

MATHEMATICS SIMPLIFIED,

AND

PRACTICALLY ILLUSTRATED.

DEFINITIONS.



A POINT is described by mathematicians, as "having neither parts nor magnitude," or, in other words, "as being more imaginary than real!" These definitions are too vague for the uninstructed, and are contrary to the conviction of the proficient. It would be as impossible for a man to fix his attention to such a minute object, as it would be to keep his eye invariably directed to one individual grain of sand in a plain; and, in fact, however ideal the situation of a point may be considered, we find ourselves on all occasions compelled to have recourse to some absolute mark, whether of a pencil, or by means of a puncture, for the purpose of identifying that particular spot, by which the more obvious parts of our work are to be regulated. Nevertheless, the artist or mechanic should proceed as closely as possible in agreement with the sense of the definition, as given by mathematicians; for, on the almost imperceptibility of the dot or puncture he may make, will not only the neatness, but even the correctness of his performance greatly depend; and the nearer such points may be to others with which they may be connected by lines, the more rigidly

should this maxim be enforced, as will be seen in the explanation given regarding the subject of lines; which will be found to require equal delicacy.

Lines are defined, as "having length without breadth," and their terminations are points. Some authors, indeed, define a line as "a succession of contiguous points;" the fact is, that lines should be drawn with such acuteness, as, like a fine hair, to be barely perceptible; for a thick line will, like a large dot or puncture, seldom fail to produce error. Such workmen as are negligent in making measures exactly, and who are in the practice of making large slovenly marks, either with chalk, or rough tools, rarely give satisfaction, and occupy more time in rectifying their first fault, than the mechanic who is attentive at the outset does in completing his job. As an early habit of exactness is of the utmost importance, and as much will, on all occasions, depend on a close attention to the minuteness of points, and the delicacy of lines, too much stress cannot be laid on these particulars: in proof of which the following example is submitted to the learner's consideration.

EXAMPLE. FIGURE I.

LET it be required to draw a straight line from the point A to the point B, and let those points be made in a negligent manner.

As slovenly persons are commonly found to be very consistent in that characteristic, we may reasonably conclude that the points will be made amply extensive, and that the line will be drawn rather at random, instead of being carried with the utmost care from the center of the point A, to the center of the point B. Now, if this probability is admitted, we shall see the origin of such a wild system of deviation, as cannot fail to convince the atten-

tive spectator regarding the result of such a want of correctness. The points A and B, in lieu of being nearly imperceptible, are absolutely measureable, and, from their magnitude, open the door to all the evil consequences attendant on a want of precision. It is obvious the line C D should pass through the centers of the points A and B; but as it has an obliquity, owing to the extent of those points, which are nevertheless cut, and afford an excuse, though a very bad one, to the indolent mechanic, the work must be very incorrect. This kind of error is unpardonable on all occasions; but especially in the setting off, or measurement of an angle, where the deviation from the proper direction would produce the error, as pointed out in the line E F, and make a difference of two degrees and more. The impossibility of performing work under such circumstances, must be obvious, and justifies what has been advanced here regarding matters which have been treated with much apparent insignificance by all the authors whose works have fallen under my observation.

Lines may be either straight, curved, or irregular.

A Straight Line is that which is drawn by the shortest way from one point to another; and too much care cannot be taken, when the points are strongly marked, that such line should pass exactly to, or through their centers. Fig. 2.

Curved Lines, that is to say, such as have a regular curve, are generally parts of a circle, and would, if continued with the same degree of curve, meet and complete a circle, like the arches of most bridges, bows, and similar forms. Fig. 3.

Irregular Lines are those which have not any particular object, and whose courses are uncertain; such as the windings of roads, the appearance of a piece of string thrown negligently on a table, &c. Fig. 4.

Parallel Lines, which may be two, or more, are such,

as from their situation can neither meet nor retire from each other. Thus, if two or more circles be drawn from the same center, they will be parallel to each other mutually, for they must invariably preserve the same distance at which they commence. Parallel lines may be either straight, (Fig. 5) curved, (Fig. 6) or irregular, (Fig. 7). Probably we cannot give a better idea regarding them, than by referring to the track made by two wheels of equal size, placed at the ends of the same axle, as in a coach or curricule, where it will be seen that both perform the same course, and invariably preserve their distance from each other, notwithstanding the various turns made by the carriage. Such lines are called parallel; but they are only so during the time they may retain their original motion or distance: hence parallel lines cannot possibly form any angle with each other, for they never can be brought to the same points.

A Superficies, or Surface, is always supposed to be level, unless otherwise described, and is bounded by lines, or sides, which may be either straight, curved, or irregular. No surface can have fewer than three sides, unless partly circular, but may have any further number at pleasure. Take three pieces of thread, &c. and place them on a table, so that their several ends may meet each other, and a three-sided figure will be seen, called a triangle: the spaces thus inclosed by the threads, will be called a superficies, or surface. Fig. 8.

A Surface has length and breadth, but no thickness; and, in this way, we, generally speaking, estimate the extent of meadow, where we measure the number of acres on the surface of the land, without considering that the proprietor possesses all under that surface, down to the center of the earth. Fig. 9 and 10.

Surfaces are also denominated Planes.

A Solid has length, breadth, and thickness, such as

dice, a log of wood, a chest, or any other object which can be measured in three different directions; even the thinnest sheet of paper is a solid; for, though its own thickness can scarcely be ascertained, yet many, put one over the other, as in a quire, become, very obviously, measurable. A box, or chest, though empty, is considered a solid; the term does not apply in the common use of geometry to absolute density, as is understood regarding the quantity of timber contained in a beam: it is sufficient that the object contains length, breadth, and thickness, to deem it a solid.

As a superficies, or surface, is bounded by lines, so a solid is bounded by surfaces, such as the top, bottom, sides, &c. of a chest. No solid can be formed with fewer than four surfaces, but it may have any number at pleasure, and may be of any form, whether like a cross, a pyramid, a horse-shoe, a table, or any article possessing substance. In this particular we relinquish the ideal properties of points and lines for that reality which exists only where measurement or weight can be ascertained or estimated. Fig. 11, 12, 13, and 14.

The Circle occurs so often in the course of this work, and forms so principal a feature in mathematics, that we cannot be too attentive to its figure and properties.

A Circle, Fig. 15, is formed by the continuation of a line from any given point, as A, which, commencing with the smallest possible deviation, as A D, from a direct, or straight course, A C, continues in that bias, so as always to preserve exactly the same distance from another given point, E, within the curve it thus describes, and ultimately, by its uniform inclination to a circular course, comes round to the same point, A, from which it originated. The point round which it revolves, is called its center, and, as has been before observed, though marked as it cannot fail to be by the compasses when drawing the

circle, must be considered as extremely minute. The line thus drawn is called the circumference of the circle, and is always supposed to be divided into 360 equal parts, called degrees (marked $^{\circ}$), and each degree is supposed to have 60 equal divisions, called minutes (marked $'$), and each minute to contain 60 seconds (marked $''$); however, little use is made of the two last divisions, degrees with their halves and quarters being generally found equal to common purposes. Fig. 16. The circle is often divided into two equal parts, by a line, C B, called the diameter, which must pass through its center; these equal parts are called semicircles, and contain, of course, 180 degrees each; or the circle is divided into four equal parts, called quadrants, by means of two lines, both passing through its center, and forming the figure of a cross; each quadrant contains 90 degrees. The degrees marked, or understood to be on the circumference of the circle, are all computed from its center; they are of the utmost importance, and if marked, should be done with the most scrupulous attention to perfect equality. Two lines drawn from the center through any two divisions on the circumference of the circle, are called an angle, the measurement of which is ascertained by the number of degrees that stand between them: thus, if a line cut the circle at 62, and another at 75, they will form an angle of 13 degrees.

From what has been above described, it will be evident to the learner, that the center is equidistant from every part of the circumference of a circle, and that a line, called the radius, from the center to any part of the circle, would every where measure alike. The radius is half the diameter of a circle; that is to say, half of any line drawn through its center, from one side to another, as A B.

Portions of circles are called Segments, and a right

Line drawn from one point of any segment to the other, as from B to D, like the string of a bow, is called the chord, and the segment is occasionally called an arc.

No circles can have the same center, but such as have their circumferences parallel.

Angles are formed by the crossing of two, or more lines, or, by their meeting at the same point. Such angles as are sharp, like a wedge, are called acute, and this term applies to all that do not measure 90 degrees; that is to say, such as do not amount to a right angle, or perfect quadrant, as the corner of a square, table, &c.

A Right Angle always measures exactly 90 degrees, and causes the two lines of which it is formed to stand perpendicular to each other. By the word *perpendicular* we are not to suppose all lines so called to be upright, as the wall of a house, &c. but that they would appear so if the line, or object, to which they are said to be perpendicular, were turned full to our view, parallel to the line of our eyes. Thus, if four lines be drawn through the center of a circle, dividing that circle into eight equal parts, each of these lines, however oblique many of them may appear, will be perpendicular to such other as may be cut thereby at right angles, so as to measure 90 degrees on each side, and that the two points, or ends of such two lines, may measure one-fourth of the circle. If there be eight spokes in a wheel, the first will be perpendicular to the third, the second to the fourth, and so alternately all the way round, without reference to the position of the wheel, whether standing or laying on the ground. Fig. 17.

Obtuse Angles are such as occupy more than a right angle, and consequently contain more than 90 degrees, but must be, though in the smallest proportion, less than 180 degrees; consequently measuring more than a quarter, but less than a half circle.

In describing angles by letters. as ABC, the middle

letter, B, ever applies to the apex, or junction of the lines, and is called the angular letter; the other letters exhibit the lines, which by their meeting form the angle.

N.B. The Triangle (Fig. 8) is formed of three given lines, A, B, C; the angle where B stands, would be denoted by writing A B C; that where C stands, would be written B C A; and that where A stands, would be written C A B: the latter would be said to be formed by the lines B A and C A. In the 15th Problem this figure is more particularly noticed.

The Square is that figure which has four equal sides, and of which all the four corners are right angles; hence the opposite sides must inevitably be parallel, and each side must be perpendicular to its neighbours. A square implies only a surface, and has no relation to thickness; (Fig. 20) if, however, it be as thick in every part as it is long and broad, it then becomes a cube, which in every part measures the same, and whose corners are all right angles, like dice, &c. (Fig. 21.) Of this more will be said hereafter.

This requires to be well understood, for it is the basis of innumerable calculations in arithmetic, under the designation of "square root," and "cube root."

A Rhombus, or Lozenge, is a figure which has four equal sides, but whose corners, or angles, differ; two being acute, and two being obtuse. The shape of diamonds, in playing cards, is a rhombus. The acute, and the obtuse angles are opposite to each other mutually. Fig. 22, 23.

A Rhomboid differs from a rhombus in having its opposite sides, and its opposite angles, equal: of course this figure may be oblong, and appear sloping, or inclined.

A Parallelogram is the same in regard to parallels as a square, but it is longer than it is broad. Such is the shape of a playing-card, the length being greater or less at pleasure. Fig. 24, 25.

Every other four-sided figure is called a Trapezium, and may have all the sides and angles unequal. Fig. 26, 27,

a Trapezoid has two opposite sides parallel

The various Constructions of Three-sided Figures, called Triangles.

The Equilateral Triangle is formed of three sides, all of which are equal, and which, consequently, must cause all the corners, or angles, to be equal, each measuring 60 degrees internally; or 120 degrees each, if measured externally. Fig. 28.

The Isosceles Triangle has greater length than breadth, and resembles a pair of compasses opened to any angle under 60 degrees, both legs being of the same length, but the base, or third side, varying as to extent. In the Isosceles triangle the two angles next to the base are equal. Fig. 29.

A Scalene Triangle has all its sides unequal, and, of course, none of its angles can be equal. Fig. 30.

A Right Angled Triangle is a most important figure, and is so called, because one of its corners is a right angle; two of its sides may be equal, but the third side, namely, that which is not connected with the right angled corner, must be longer than the two others; it is called the hypotenuse. Fig. 31, 32.

The base of every figure is that part which presents itself to us in a horizontal position, as a line drawn across a page from one side to the other, and the altitude is applied to the height of the figure in our upright position, as from the bottom to the top of a page: in sloping, or inclined figures, that side which is most inclined to the horizontal position, is considered as the base. But in most prolonged figures, such as parallelopipedons, prisms, &c. which have solidity, the terminations are

called bases; thus the top and bottom of a large squared timber would be so termed. Fig. 33, 34.

Compound Figures are such as have their outlines mixed, some being straight and others curved. Here it should be understood that all irregular figures and lines are composed of straight and of curved lines; for there cannot be any crooked line whose parts, taken separately, do not apply to some circle or circles, of which they may form either large or small portions. Fig. 35.

Thus a serpentine walk is composed of part of one or more circles; consequently its extent can be easily ascertained, as will be understood when the learner arrives at that part of the treatise which explains the mode of changing figures into others of equal dimensions, such as a square into a parallelogram, a circle into a square, &c.

It has been before stated, that a solid is that object which has length, breadth, and thickness; that it is bounded by surfaces or sides, and that no solid can have fewer than four sides, but that it may have any number of sides beyond four. The only right-lined figure we are acquainted with, that can be found with only three sides, is a triangular, or three-sided pyramid; that is to say, one whose base is a triangle, and which has three sides, all triangles, whose bases fit to that triangle which forms the base of the pyramid, and whose points meet, and form the point of the pyramid, somewhat resembling the shape of the three-sided file used for sharpening the teeth of saws, if the spike be cut away square. (Fig. 37). Pyramids may be of any number of sides, (which must be all triangles), provided their bases have an equal number; and these triangles, if similar to each other, must be acute. Pyramids are, in general, found with four sides, resting on a square base, especially those intended for ornament, such as the Obelisk in St. George's-fields, and various

other monuments to be seen in public places.—N. B. In these the base or bottom is not considered as a side. Fig. 38, 39, 40.

Another figure may be produced from four sides, but it must be of a mixed nature, such as half a cheese. Fig. 36.

A Cone is a solid which differs from a pyramid in having its base a circle, and in being, consequently, of a circular form all the way from the base to the point. Loaves of sugar may, generally speaking, be termed cones, though not properly conical. A true cone is created by the revolving, or spinning of a triangle on its center, supposing the point to serve as a pivot, and another point to be in the center of the opposite side. Some authors admit that cones may have oval bases, and be oval to their summits; but such cannot be deemed perfect, for it must be obvious that they never could come to a regular point. The only cone acknowledged in mathematics, is that generated by the motion of a triangle, supposing its apex, or upper point, to be one pivot, and another pivot to be in the center of its base; thus, in the triangle B A C, (Fig. 42) the pivots would be A and D. If the triangle were to be turned round on them, the space occupied by its revolution would be a cone, or sugar-loaf, of a perfect form. Fig. 41.

A Cylinder is a figure whose base and top are parallel and equal circles, and whose sides are perpendicular thereto, forming a shaft like a round ruler, or a rolling-stone, every where of equal thickness. Fig. 44.

A Sphere, or Globe, is a solid figure which, however taken, always gives the same girth and the same diameter, such as common shot, &c. Fig. 43.

A Prism is a solid figure, having its base and top alike, equal, and parallel; it may have any number of sides not fewer than three, all of which are either squares or parallelograms, but generally the latter: thus a pedestal, having a number of sides, is a prism. Fig. 47, 48.

A Parallelopipedon is a prism whose bases are squares, or rhombuses, and whose four sides are parallelograms: thus a timber which is square is a parallelopiped. To constitute a true parallelopiped, the bases, or ends, and the opposite sides must be reciprocally parallels. Fig. 45, 46.

On the foregoing Definitions depend the whole of the following propositions; I trust they are delivered in such unequivocal and simple language as to require no further explanation; I shall, therefore, proceed to the more interesting and important parts of the subject.

ADVERTISEMENT.

To facilitate the understanding of many matters that follow in the course of this work, it is proper to state, that figures having many sides are termed polygons, and are distinguished thus:

A figure with 5 sides, is called a pentagon.

6 ————— hexagon.

7 ————— heptagon.

8 ————— octagon.

9 ————— enneagon.

10 ————— decagon.

11 ————— endecagon.

12 ————— dodecagon.

15 ————— quindecagon.

PROBLEM I.

To draw a Right Line between two given Points.

PLACE your ruler, or scale, very carefully, within such distance from the points A and B, that when the pencil, or pen (which should have a fine point, and be used very lightly) is applied, it may pass in the most even manner from the center A to the center B.

APPLICATION.

So much has been said on this subject, that it will be sufficient to remark in this place, that, unless great attention be paid to drawing a right line with correctness, neither the builder, the sawyer, the smith, the carpenter, the joiner, nor any other mechanic can possibly execute his work with neatness, celerity, or accuracy. As to the finer branches of business, as in modelling, architectural design, engraving, and a thousand other similar pursuits, the utmost precision is needful; so much so, indeed, that we ordinarily find persons following such professions, in the habit of using spectacles, or glasses with great magnifying powers: the keenness with which they are obliged to view their work, especially by candle-light, renders such an aid indispensably necessary, at least to a majority.

PROBLEM II.

To prolong a Right Line.

HAVING occasion to add more to the line AB than the length of your scale can cover, to insure accuracy,

it is needful to overlap a part of the original line, say from C to B, and to carry it on to D by means of that part of your scale which goes beyond it; repeating this as often as may be necessary to produce the required length. Prob. I. Plate 2.

APPLICATION.

In surveying, where a theodolite, or other instrument can be used, the extension is easily managed by the line of sight; but, in small work, or where such an aid cannot be obtained, the method pointed out in this problem must be resorted to.

While treating of the rule for prolongation, we necessarily refer to a scale, &c. as applicable to the size of the paper on which the work is to be laid down; but in the field, a cord, &c. must be applied in the same manner as a scale is used on paper. A person wishing to build a long wall (such as for a park or garden), not having a line of sufficient length to proceed the whole extent of any straight direction of the work, must prolong his line according to the direction that follows. Having marked with a spade, &c. along the first range of his line, he should fix a pin at about a third part from that end which is to lead forward, and having another pin at its first termination, the other end of the line should be brought up to the first-mentioned pin, or stake, and the leading end being carried on, the person stretching the line should look back, and move to the right or left, until the cord to be laid down touches both pins, and brings both lines into one direction. When a cord is not at hand, or cannot be used, on account of water or roads intervening, a line of poles will be found very correct; the two first poles give the direction, and as each succeeding pole is set up, either the person standing at the first pole, or he who fixes the last, should be careful that in looking along

the line of poles, only that next to the one he stands at, be seen. If this be done correctly, the line will infallibly be true; but, if a third pole be seen from either spot, especially if the spot whereon it is fixed can be distinguished, there must be a deviation from a straight line. The top of a pole which is not set up perpendicularly, may, perhaps, be seen, though the spot where it is fixed may be precisely in the given direction; therefore attention should be paid, by means of a line and plumb, to fixing at least the two first exactly perpendicular, for if they have a bias, it will be difficult to establish the places of the rest. In undulating ground, this is an excellent method.

This problem merits the attention of those who have the formation or conducting of roads, or who have to partition off large portions of land. Even the ploughman will find this problem aid him in drawing straight ridges, and in directing various operations within the ordinary practices of husbandry.

PROBLEM III.

To divide a Right Line into Two equal Parts.

OPEN your compasses the whole length of the line A B, and placing one foot at A, carry the other round in a sweep, then fix the foot at B, and make another segment opposite to the first; a line drawn from the point D, where the segments cross above the line A B, to the point C below it, will not only divide the line A B into two equal parts, but will produce a perpendicular falling on its center. Prob. II. Plate 2.

APPLICATION.

This operation is of such frequency in almost every line of business, that it cannot be too much studied. It

is common to find workmen measuring with a string, and doubling it, to find the center of a plank, for instance; these persons think they strain equally at the line, both while they measure the whole length, and its half; but they rarely do so even with a single line twice successively, and they make no allowance both for the less liability of a shorter end to stretch, and for the greater degree of force required to extend a double cord. If a line can be drawn along the part to be divided, let a pair of good compasses measure off equal numbers of strides from each end, until they leave but a small space undivided near the center; when, by adopting this mode, not only will the plank be correctly divided, but if the line has been drawn down the middle of its length, a perpendicular will be furnished, enabling the workman to cut or fit the plank at exact right angles. In building, a rule or staff of many feet will supersede the use of compasses; and where precision is needful on a large scale, the work may be done by means of a chain, &c. whereby accuracy may in general be insured. In planning, or in measuring off on paper, this operation will be found highly correct, and far more speedy than the bungling mode in use among novices, who expend much time in their endeavours to portion off a half by means of common compasses: indeed, even proportional compasses must be more than ordinarily correct if they give a true division.

PROBLEM IV.

To erect a Perpendicular at the End of any given Line.

OPEN your compasses, and place one foot at the given point A, then fix the other foot at any distance above the line, as at B, from which, as from a center, draw a circle

with the foot placed at A. From the point C, where the circle cuts the line, draw the oblique line CD through the center of the circle, and the line DA will be perpendicular to CA.

APPLICATION.

The carpenter and mason usually have recourse to the square and bevil, which, certainly, for ordinary purposes are sufficiently correct; but, in the finer arts, as well as in particular instances among the various classes of persons concerned in building, &c. the utmost precision is necessary.

A deviation from an exact square, or right angle, however trifling in the commencement, leads to the most distressing errors, and infallibly produces a great difference, which becomes gradually more obvious as the work is extended. In fact, there is not any part of measurement so common, or so much requiring attention, as the formation of a right angle; joiner's work, the construction of all the wood-work of a house, of a cart, or waggon, and of almost every kind of machine, depend chiefly on this problem. Not even the bevil should be trusted where fine work is to be produced; at least, if, in the first instance, the bevil be used while making out the rough pieces, great care should be taken afterwards in correcting any improper bias. The smallest error in one side of a square, supposing all the others to be faithfully at right angles, must generate a trapezium: this may be fully understood by an examination of the 2d Fig. where, at A, a false, or acute, angle is made in the outset, but all the others are correct. The original error gives all the succeeding work a wrong direction, and occasions one side to be shorter than the others, there being a section of it overlapping at B, from which it also takes off a small part. Fig. 3 exhibits the effect produced by an excess

in setting off one of the angles, whereby the fourth side will gape, or stand apart from the first side at C, owing to the error at D.

PROBLEM V.

To divide a Line into any Number of equal Parts.

At the end of the line A B, which is to be divided, say into four equal parts, raise the perpendicular A C (Prob. IV.) to any extent that may be necessary; then, from your scale, measure off with your compasses four inches, or halves, or quarters of inches, or other space proportioned to the size of your work, and placing one foot of your compasses at B, carry the other foot to such part of the perpendicular, A C, as it may fall upon. Draw the line B C, and mark off the equal parts from your scale thereon. Repeat the whole of the foregoing process from the other end of the line to be divided, and from the equal parts in the first oblique line B C, draw lines to the opposite divisions on the oblique line A D; they will divide the given line A B into four equal parts.

In this way a line may also be divided as any other given line is divided, for the proportions will be perfect; but, if the given line be the shortest, then the line to be divided must be diagonal, or oblique, and perpendiculars must be raised from the respective parts in the given line.

APPLICATION.

This problem is of particular service, as it enables us to divide any given line into any number of equal parts, and that too by means so extremely easy and correct. By this device we are in possession of a method of reduc-

ing, or of increasing a scale of parts at pleasure; for, having a given scale of any moderate extent, it is easy to transfer all its admeasurements, or divisions, into whole parts and fractions, with the greatest precision. This is often of much moment, especially to architects, &c. who have to reduce the plans and designs of others to a scale fitting their own purposes, or to be conjoined with work doing by another scale. We learn also by this means one method, not indeed much in use, of constructing a parallel; for having once obtained the angle CBA , we can, without more knowledge than is afforded by the preceding problems, draw the opposite angle DAB ; when the lines DB and AC must be parallels. Those who understand the use of the scale of degrees on a protractor, sufficiently to draw a perpendicular by that convenient instrument, can solve this problem by means of only one line of equal parts, drawing lines from the points 1, 2, 3, 4, in the oblique scale, which fall perpendicular on the straight line AB , by which the protractor is guided.

In pitched roofs, whose uprights are to sustain rafters, the corresponding mortices both in the rafters and in the beam supporting the upright, are thus accurately given; for the divisions, both on the straight, and on the oblique line, will bear a just proportion to each other throughout.

PROBLEM VI.

To draw a Parallel to a given Line.

FROM two points, E, F , near the extremities of the line AB , with an opening of your compasses, corresponding with the distance intended for the parallel to be made, draw two equal semicircles or segments. A ruler or scale applied so as just to touch the parts G, H ,

which are most remote from the given line, will enable you to draw the parallel CD ; or it may be done by raising two perpendiculars (Prob. IV.) viz. one at each end of the line AB , and measuring off on each of them at CD , the distance at which the parallel is to be drawn; the line drawn from CD will be that parallel.

APPLICATION.

This offers the most simple means, such as even in fine work cannot be excelled, but by the application of a parallel ruler, many of which are, however, greatly defective, and inferior to Marquois's triangular scale. On an extensive plan this mode is certainly to be preferred, being simple, and sufficiently correct. The drawing of a parallel for the opposite faces of buildings, &c. may be accomplished by means of a cord, or rather by a pole, very readily. On some occasions repeated parallels must be made, one beyond the other, to reach any distant object.

The 2nd method is more minute, and consequently more objectionable, where great nicety is required; it is given as applicable, perhaps, to some cases, but I know of few where the first mode may not be practised with equal success. Certainly, if it were required to draw a very distant parallel, and at the same time give the sides of an area, I should give the preference to the second mode, as performing both operations at once. Those who have to saw beams into planks, or stones into slabs, have an easy mode of so doing, by measuring off from the smoothed edge any number of distances from each other, on two perpendiculars drawn across, near, or at the two ends: say, that a log is to be cut into six planks of two inches each in thickness; if one side be well levelled, it affords the standard whereby to square two others, on which the

work is to be done. If the original points are fairly set off on the upper and lower sides, they must be parallel to each other, and to the directing side, even though the two ends should not be right angles, nor equiangular thereto, as in Fig. 3.

PROBLEM VII.

To make an Angle equal to any Number of Degrees.

MEASURE off 60 degrees with your compasses, from the line of chords * on your scale, and applying one foot to A, with the other describe the segment B C, on which segment measure off the number of degrees, say 45, required for the angle from the line of chords also; then draw a line from A to the point D, where the segment was marked by the measurement of the number of degrees, and the angle D A B will be the angle of 45 required.

N. B. Where the angle required exceeds 90° , that quantity must be first cut off by a perpendicular, from which the operation is to be afterwards performed: in this way any angle may be formed with the greatest ease and certainty, and any increase or diminution may be made.

APPLICATION.

The most simple mode is certainly from the line of chords, if the scale has one marked on it; but, if only a quadrant, or other simply graduated instrument be at hand, recourse must be had to setting off the number of degrees from its edge. Place the point A, Fig. 2, at

* The line of chords is marked *cho* on the scale.

the intended center, and carry the lower edge A B along the direction in which one line of the angle of 45° is to proceed; look to the edge of the quadrant, and count 45° ; then with a fine pencil, or needle, &c. make a mark, and from the original point, A, draw the line A C, which will make the angle B A C of the required extent. Whether the line be long or short, or of unequal length, still the value of the angle remains unchanged.

This problem teaches us either to form new work, according to any particular calculation, or to imitate what may have been already done. In surveying it is a principal operation, for on the accuracy with which the angles are taken by the sight-instruments, and reduced to paper, will the correctness of the performance principally depend, as has already been exhibited while treating of the formation of a square.

In pitching roofs according to a plan, we must look to this problem for a guide; as also in the ascent of stairs, and a thousand matters in which one part depends on the other; in fact, we scarcely see any operation in mechanics wherein it is not employed.

To extend from the paper to the scale of practice, we have nothing to do but to ascertain the lengths of the three sides of a triangle, observing to measure minutely; the proportions on the small scale will direct for the greater; for, if the three sides be on paper, 3, 6, and 8 lines, and that lines represent feet, then in the work these sides will be 3, 6, and 8 feet respectively; and we reduce from the great work to the paper by exactly the reverse. As to fractions, such as tenths or twelfths, they must be taken by means of the diagonal at the end of the scale, with the utmost nicety. The diagonals being parallels, A, A, &c. will cut the horizontal line b b into regular proportional parts: thus ten parallels will give tenths for fractions, and twelve will give twelfths. And

this scale, which is, perhaps, unequalled for accuracy, may be constructed in a minute; the horizontals may be more or less distant than the diagonals, but they must be equidistant, and parallel among themselves. The number of diagonals must be equal to the number of horizontals, and may vary according to any number of equal parts required; thus, for ten equal parts there must be eleven of each kind of lines; for twelve, there must be thirteen; for eighteen, there must be nineteen; and so for any number, there being always one more line than there are equal parts. Fig. 3 and 4.

PROBLEM VIII.

To erect a Perpendicular on any given Point in a Line.

FROM the point A, in the line BE, measure off any convenient spaces, D and E, equally to the right and left, with your compasses; then from E and D make segments above the line; their section, or crossing at F, will give the point from which a line drawn to A will be perpendicular to the line BE.

APPLICATION.

To reduce this very important problem to practice, such as for erecting a pillar, &c. perfectly perpendicular (which is not always done by a plumb), we must, in lieu of the segments, take two battens, or ends of wood, &c. of equal length, fixing their bases at the given points D and E, their union at F will give a true perpendicular to the point A, where the center of the pillar should rest. It should be observed, that situations may occur in which a center must be ascertained, but where a plumb could

not be used; in such a case, the mode above described will be found equal to every purpose, as a perpendicular may be made to correspond with the occult central point. Let us, for instance, place an oblique cone, or sugar-loaf, on a board, and seek for its center; to do this, measure off equal distances from the base, as A to B, and from C to D, in Fig. 2, and the meeting of two lines, or battens, at E, which have one end fixed at B and D, will give the perpendicular that points immediately to the center of the cone: and it is the same, even if the cone be rested on the edge of its base, as in Fig. 3; draw a perpendicular from the raised rim, F, to the plane at G, and measure equal distances from G and H; resting-places, K L, for the ends of the battens will thus be given; for the center of any object must always be over the center of the place, or extent that it covers, else the division of a line into similar parts, as shewn in Prob. V. would be false. Any number of perpendiculars may be thus raised from any number of points on a given line; one serving as a standard for the formation of parallels, all of which must be perpendicular to the same line. Where an object occupies a space unequally, as in Fig. 4, we must find the center of the board on which it stands, by forming rectangles either around its base, or over its altitude, as in Fig. 5.

These cases occur so frequently among artizans, that the modes of obtaining centres of lines, or spaces, especially on broken, or uneven surfaces, cannot be too much studied. It is obvious, that, when we measure uneven surfaces, we never can be exact, if the surface be our guide; but by a series of rectangles we avoid the irregularity, and accomplish our design with ease and certainty. In Fig. 6 we see the impossibility of ascertaining a perpendicular on any given part, except by means of a base line, either real, or imaginary, as from A to B; by per-

pendiculars at those points, as $A C$, $B D$, we are enabled to draw $C D$ parallel to the base, and from that parallel, as the base is inaccessible, we have our various perpendiculars at pleasure, as at $E F G$, &c. If we would ascertain the spot on which a perpendicular would fall, if set off from a right line, $A D$, having an inclination from the horizontal, we have but to measure off from the oblique line such a proportion as we would cut off from the horizontal, $A D$, and after measuring the angle $D A C$, say 35° , return another angle, $A D B$, of 55° for the perpendicular, and the required spot, B , is touched; for the oblique and horizontal lines will be cut in the same proportion by the second angle, whose measurement, added to the former angle, that is $55+35$, make the complement $A B D$ of the right angle found between the horizontal line and the perpendicular. See Fig. 7.

This is also founded on the 5th problem, and it should be remarked, that the whole of this, and, indeed, of the foregoing problems, may be computed on paper, and afterwards be worked on a larger scale by the artizan; the former will prove an exact imitation of the latter.

PROBLEM IX.

To cut a given Angle into Two Parts.

LET it be required to divide the angle $B A C$. Set off with your compasses any convenient length, as D and E , on the lines $A B$ and $A C$, measuring from A ; then draw the line $D E$, on which form the equilateral triangle DEF , by making segments from D and E meeting in F . A line drawn from A to the point F , in the said equilateral triangle, will divide the angle $B A C$ into two equal parts.

APPLICATION.

This problem occurs in daily practice among all classes of artizans and labourers; it is, in fact, dividing the circumference of a circle, as well as the angle; for if we consider the angle, or wedge, as a portion of a circle, or of a polygon, we must necessarily suppose such figure to be divided as we divide any of its parts: thus, let it be required to cut the angle BAC , Fig. 2, into two equal parts.

Now, that angle being 72° , is the fifth of a circle; consequently, as we divide it into two angles, each measuring 36° , we, by the same operation, are actually dividing the circle into ten equal portions.

When angles thus form equal parts of circles, such as 10° , 12° , or any number of which they are multiples, the operation may be performed at once by the protractor. Mathematicians always consider the whole figure as at command, but the workman will often find a large portion of it inaccessible, as in a broken pyramid, Fig. 3.

For such a case the mathematician makes no provision. To bisect such an angle with precision, let two or more lines, as AB , CD , be drawn on either side of the pyramid; bisect these lines, and the line EF will pass in such a direction as must divide the angle exactly. The same may be done on every side, if required. The lines may be in any direction, at pleasure; all will depend on the accuracy of the bisections. The base may serve for one line, but it is best to preserve it as a proof of the work being correct; for if it be so, the base will be divided into perfectly equal portions. It is also to be considered that the ends of the pyramid may both be mutilated, in which case they could not be used as guides.

In constructing bridges, we are compelled, from the

utter impossibility of describing arches from their centers, on account of those centers frequently being below even the bed of the water, to build according to a scale, or, in other words, by imitation; in such cases it is necessary to draw a line above the level of the stream, and from certain parts to measure off corresponding degrees on corresponding angles. Where a frame cannot be constructed of a true form, we are compelled to resort to this method for perfecting the curve of the arch, especially where a true level cannot be found, whereon to trace the segment as a guide for laying the frame.

The wheelwright must comprehend how the angles of the spokes are to be divided as to distance, and even the plaisterer who ornaments friezes, capitals, &c. will feel himself at a loss to proceed with regularity, unless he may be expert in this very simple part of the science, on which, however, much of the sublimer branches essentially depend.

In building a truss-roof, the angles must all be divided, as may be seen in the following design, where only three pieces are used, and give great support to the couple; being well bound together by a strap and pin at the center O; or in a wooden cupola, to be covered by a leaded roof, as on the left side, where, by pointing all the supports to the center, the pressure is every where alike: in short, we could adduce innumerable instances where the division of an angle is necessary to be understood, both in useful and in ornamental works. Fig. 4 and 5.

N. B. On the right side, the rafter is equally divided by three supports, by which means the pressure is uneven on the inner frame, or cupola.

PROBLEM X.

To draw a Perpendicular on a given Line from a given Point out of that Line.

FORM the segment AB, from the given point C, on the given line DE, bisect, or cut in two (Prob. III.) the chord FG in H, and the line joining C to H will be the required perpendicular.

APPLICATION.

The manner in which this may be done on paper with the greatest facility, is thus given; but we may, on this occasion, again revert to the 5th problem, and also to the last part of the definition of the 8th, where it may be seen that a perpendicular may be efficiently drawn from any part of an inclined line to any given line. The protractor does all this at once, but we must not conclude that such an aid is ever at hand. A rule, serving as a scale, and a pair of compasses for setting off distances, and for describing circles or segments, are generally as much as we can command, therefore the directions above given are most importantly useful.

In building, this problem is often required, without the work-people being sensible of its facility, or obvious necessity. We daily see men setting up the couples of roofs in a tedious, bungling way, while, by means of a chalked line on the floor, running under the range of the summits of the couples, and a plumb dropped from them to that chalked line, the true position of the couples would be given; for if the walls be of equal height, and their distance be preserved, or at least be alike from the

bottom to the top, the plumb-line must divide the space equally, and ascertain the true center. Let us, however, suppose that the walls should be of unequal height, as in Fig. 2; the uninformed workman would find it difficult to pitch the roof so that the angles on each side should be equal. He has only to draw the line CD parallel to AB , and the center of that line will be the point at which a plumb-line dropped from E , will, when the angles CEF and FED are equal, cross it.

Let it be required to erect a pillar, whose sides have a graceful swell, or a pyramid; it is easy to adjust all the pieces, so as to fit exactly, previous to their being carried up to the situation in which they are to be fixed; but, while placing them, the center must be carefully kept in view, else the edifice will be disfigured, or eventually become dangerous. For this purpose, a simple apparatus suffices; (Fig. 3) a projection of frame-work, A, B, C, D , either square, or circular, that is either a pyramid, or a cone, having, if the work be intricate, spirit-levels in two of its bases, surrounds the work at a small distance, and rises a moderate height, to E , say 6 or 8 feet, above the level of the work; by means of a pulley, F , at the top, a plumb, G , is suspended, and raised as the work proceeds; the sides being governed by admeasurement, while the perpendicular is preserved by means of the plumb, which invariably points to the center, H . When the work advances, so as to require the frame to be raised, the center is marked on the surface, and serves to direct in replacing the frame with accuracy, after its being carried up to its new situation, and thus successively to any height. This is also applicable to stairs winding round a pillar, where the center must be carefully preserved.

I recollect seeing, what would otherwise have been a handsome steeple, built all awry, merely because the architect had not recourse to this kind of frame; he used a

gibbet-plumb, which, owing to its want of firmness, perpetually shifted, and caused every layer of stones to have a distinct center. This fact has often occasioned much amusement to the inhabitants of Calcutta.

PROBLEM XI.

To make a Square.

THE mode of doing this with precision, is to erect perpendiculars at A and B, according to Problem IV. making them exactly as long as your base, by prolonging them to E and F; then draw a line from E to F, and the square is completed. So much, however, generally depends on the perfection of squares, that it would be well to examine, by means of the line of chords (as in Problem VII.) whether the angles B A E and A B F be exactly 45° .

APPLICATION.

Here we have one of the most useful, as well as one of the most difficult operations in mathematics; it requires the utmost nicety to describe this figure, which is the parent, or the proof of at least half the problems in use. The truth is, that it requires a nice eye and a steady hand to measure any angle with exactness; fortunately, we have the means to correct, or at least discover, any error in the formation of a square, since the uniformity of its angles and its sides gives a directing standard; for if the whole be not right angles, the distance between F and B will not be the same as between E and A. Again let three measures be taken from your scale, and set them off from B to S; set off four measures from B to O (Fig. 2); if the angle be a true right angle, the line, or

distance from O to S, will be exactly five such measures. It is sufficient here to announce the fact; the reasoning and proof will be found hereafter in the 47th problem of this work, which explains the celebrated 47th of Euclid's first book.

The exact formation of the square carries with it more importance than appears to the superficial observer, for, not only will the beauty of the figure itself be destroyed, but all that depends thereon will suffer in proportion. The outward and inward resistance will be greatly impaired, since the want of a true perpendicular disposes to weakness, and in too many cases occasions serious consequences. When we enter a room whose angles are not correct, we feel an unpleasant sensation; the furniture never seems to fit; we cannot sit with comfort on an uneven chair, nor at a table whose legs are not perpendicular, and whose surface is not square with the legs. This unpleasant sensation increases into an apprehension of danger, when we perceive that the walls of a house are not truly perpendicular. Sloping floors are peculiarly inconvenient, and doors that stand on frames which have an inclination either one way or the other, not only are unsightly, but are constantly out of repair.

A square, or its derivative, the parallelogram (vulgarly termed the oblong square), is either the basis, or is connected with almost every thing in use among us. Even circles, and other figures are estimated by the square, and, in general, we either deduce from, or refer to, that form, as being both in itself beautiful and perfect, or as affording a ready means of estimating, or calculating the value and contents of other bodies; be their figures ever so various. From this we may judge how correct a bevil should be in its angle, and how much depends on drawing a true parallel.

PROBLEM XII.

To construct a Parallelogram.

THE mode of doing this, resembles the manner described for the formation of a square, observing that the only difference is as to the height of the two sides, which in a parallelogram must be less than the base: with the above exception, the same references apply.

If it be an upright parallelogram, the height will of course exceed the length of the base, as in Fig. 2.

APPLICATION.

What has been said regarding the square, applies equally to the parallelogram, or oblong square; for a room of this shape, or the leaf of a table, the cover of a book, or other object, all require to be formed with right angles. The surfaces of boards for floorings, the several sides of joists, beams, rafters, &c. are generally parallelograms. They may be either erect, as the generality of doors; or horizontal, as a hearth-stone; or breadthways, as the hanging flap of a dining-table. A square cannot recline, but must always stand at right angles; but a parallelogram may lean any way, provided the opposite sides are parallel and equal, and the opposite angles similar: of this more will be said in describing Problem XXIII.

PROBLEM XIII.

To draw a Parallel to a given Line through a given Point.

FROM any part, as D, of the given line, A B, draw an oblique line to the given point, C, then measure the angle B D C, and form a similar angle, D C E: the line C E prolonged to F, will be the parallel required.

APPLICATION.

We must divest ourselves of the expectation of having always a parallel ruler at hand, and study to perform these exercises, as before stated, by means of less convenient instruments. The 6th Problem shows how to draw a parallel generally, and independent of any exact limits, whereas this problem is founded on the necessity for a parallel through a given point. The mode laid down is that generally followed by professors, but it supposes the operation to be performed by means of a protractor; if that instrument be not at hand, we must contrive to draw the parallel by means of the compasses only.

Let G H (Fig. 2) be the given line, and I the point through which a parallel is to be drawn. Draw a perpendicular, I K, touching the line H G (Problem X.) and again draw the line L M perpendicular to I K (Prob. IV.); L M will be parallel to G H. Or, according to Fig. 3, let N O be the given line, and Q the given point: draw a diagonal, from any part, N O, to Q; then with a knife, or scissars, cut a piece of paper, paste-board, &c. to fit the angle T Q N; slide the model thus cut along the line N O, and its point at Q will trace the parallel sought. This is a simple mode, and may be done without the aid of any

kind of mathematical instrument. If it be required to do this on a large scale, a line will answer the same purpose; for if three persons hold at T Q N respectively, and move as directed for the model, he who holds at Q must walk parallel to those holding at T and N; for when three lines are joined into a triangle, they form a figure, which, if kept at due tension, will not be affected by motion, or by any thing but an absolute dislocation of its parts, the perpendicular distance from Q to A being always the same. A mason's plumb-frame, for this reason, is an excellent instrument for drawing parallels within its reach, the base moving by a given line, and the parallel being drawn by a given point on the upright center.

For some occasions a cross (Fig. 4) made of two equal battens, rivetted on a point at their centers, is highly useful; the two lower points, A B, being applied to a given line, X Z, while a line passing from one upper point, E, over a small groove in the other, F, and having a plumb, G, pendant thereto, gives an excellent parallel, particularly in upright work, as in mouldings, cornices, chimney-pieces, &c. If the lower points be duly attended to, the upper parts of the cross must move parallel thereto, while the line at the top of the latter is kept tight by means of the weight.

The above simple contrivance holds forth the additional advantage of giving a vertical perpendicular, F B; and, if the guiding line, X Z, (Fig. 4) be truly horizontal, which is ascertained by the plumb, G, being in a right line with the points F and B, that perpendicular will be at exact right angles with X Z, and consequently with all its parallels. This instrument, as also the frame for preserving a center, described in the application of the 10th Problem, are of my own invention, and enabled me to execute work with great accuracy and more rapidity than by any other mode I ever saw practised. Those who have

large concerns, should have various sizes of the cross described here, so as to answer in different situations. I had one with a quadrant at the point, as in Fig. 5, whereby I could set off any angle with great readiness. Such an instrument is extremely portable, and answers many purposes.

PROBLEM XIV.

To construct a Rhombus and a Rhomboides.

THE inclination of the perfect rhombus is an angle of 45° , though it may be more or less inclined. Let the lines AB and CD of the square ABCD be obliqued to an angle of 45° , and their points be joined by the line EF, then will a true rhombus or lozenge be formed. The rhomboides differs from the rhombus as the parallelogram does from the square, only in the length of its base and its parallel. The bases of figures of this description remain fixed; that is, they are considered as being horizontal; but that point must depend on the position of the rhombus, which, like the parallelogram and square, will be occasionally found obliqued or reclined.

APPLICATION.

These figures are not of great importance in the generality of handicraft pursuits, though they sometimes occur in ornamental work. The rhombus is but a deviation from the square by means of a bias or leaning, which throwing the figure aslant, makes it assume more or less of the lozenge, or diamond-form.

The rhomboides is in the same manner derived from the parallelogram.

They do not appear to have any particular use, but as in surveying they often occur, it is proper to state that their areas, or contents may easily be found by several of the following problems, either as they stand, or by division into triangles, or by commutation into other forms.

PROBLEM XV.

To form a Triangle with Three given Lines.

TAKE either of them at pleasure for the base, say A G, then placing one foot of your compasses at A, with an opening equal to the second line, describe the segment C D, and with an opening equal to the third line, placing the foot of your compasses at G, describe the cross segment E F; their intersection at B will fix the length and direction of the second and third lines, which being drawn from the points A and G respectively, complete the triangle. It is to be observed, that to form a triangle it is absolutely necessary that the two shortest lines, when joined in length, should be longer than the third.

APPLICATION.

It has been already stated, that three given lines, when joined into a triangle, are immutable, consequently they can assume but one form. Cut a triangle from a paste-board, and after having traced its form on paper, turn it upside down, and the triangle will appear reversed, but its form must be the same; consequently its area, or internal space, can never be varied, as it may in the square, when reclined into a rhomboides. Hence we are induced, especially in surveying, navigation, &c. to adopt

the triangle, it being a fixed figure, simple in construction, and equally so in admeasurement, as the ordinary medium of calculation.

It is necessary to be understood, that when two sides of a triangle, and one of its angles are given, the third side, and the other angles, are necessarily determined thereby. Thus, if a carpenter be ordered to make a flight of seven stairs, each rising seven inches, and each of such a base as should not give to the whole more than eight feet, two inches, of horizontal extent, he would, of course, allot 14 inches to the tread, or base of each step, and seven inches for the height; but, as one angle must answer for the third side of all, individually, or collectively, he would, after measuring eight feet, two inches, on the floor, A C, (Fig. 2) erect 49 inches as the total height, D C, and his third line would be the angle of ascent, B D, as in Fig. 2. By this means he would know the exact quantity of wood required either for the steps, or their support.

The supports placed at the backs of looking-glasses, &c. intended to stand at any given angle, are on the principle of the fixed triangle, that is, of three given lines; viz. one by the glass, one by the supporting-fork behind, and one by the space between them, as placed on a table, &c. These three given lines form a triangle which must always be the same, for the three sides admit neither of contraction, nor of extension; for this reason we see the notched racks in use on the same occasions, for giving the glass more or less uprightness; but the base-line is of course increased or diminished, in proportion as the fork is brought more or less forward; hence the angle at the foot of the glass may become a right angle, but never can be obtuse, whilst the angle at the junction of the fork with the back must always be acute: that at the top of the glass will be generally acute, but may be an obtuse, or a right angle, as the glass is made to slope backward.

In roofing a house, we generally proceed by some plan, or previous disposition of the work; hence all the parts are duly provided for, and, being once regulated on paper, may be easily executed. If, for instance, we would have a regularly-pitched roof, the joist will be as the base of an isosceles triangle, and the two rafters being each more than half as long as the joist, will complete a triangle, whose altitude, or pitch, will be determined by their length. (Fig. 3). If one rafter be longer than the other, the roof will be irregular, as one of its sides will be more upright than the other, as in Fig. 4, unless (as in the application of the 10th Problem) one of the walls, or supports, be so much lower than the other, as to require an additional length of rafter to reach it.

Persons who have to make any thing of a triangular-form in large quantities, have but to cut out their work, as it were, wholesale, in three separate lengths, as in a rack for arms, &c.; and by taking for each, one of each length, or parcel, they cannot fail of producing a similar and equal triangle throughout.

PROBLEM XVI.

To find the Center of a given Circle.

DRAW a right line, A B, any how through the circle, and bisect the line in C, and draw the line D E through C, perpendicular to A B. If the line D E be bisected, the point F, at which it is halved, will be the center of the given circle.

APPLICATION.

It is utterly impossible to perform any regular work, either within, or about a circle, unless its center be pre-

viously ascertained; that once known, enables the artist or mechanic to regulate all the divisions with precision, and with the utmost facility.

It often happens that arches, &c. are partially injured, so as to require considerable repair; certainly, a person with a correct eye may do much towards restoring the work to its original form, but it cannot be expected even of the most experienced, that unerring precision should follow every attempt, if the center were not previously discovered. Let us suppose that a timber is to be bored through with an instrument of a certain diameter, and that there are already fissures or blemishes which it is of importance to include in the boring, as in Fig. 2.

It is obvious, that unless their center be found, the proposed advantage cannot be taken; or say, that certain ornaments are to be exactly arranged within a given circumference, or that a new wheel for a clock have not its center marked or pierced, how is the ornament to be regularly disposed, or the wheel to move in an exact manner, unless their centers be correctly ascertained?

The manner of performing the process laid down in this problem, requires the most scrupulous nicety, especially in small circles; on which account it is often prudent to prove the accuracy of the work by a second chord, as at DB, which having a perpendicular drawn through its bisection at G, cannot fail to cut the former line, DE, in F; which consequently must be the center of the circle.

Another way, though not so certain as the foregoing, offers for ascertaining the centers of large circles with sufficient precision for many purposes; let one point of your compasses be fixed at E, open to such a distance as may just come in with the line of the circle, when, taking a sweep, as from O to P, mark the spot at which the seg-

ment OP blends perfectly with the circle at D , and drawing the line DE , bisect it in F . See Fig. 3.

PROBLEM XVII.

A Segment of a Circle being given, to find the Center from which that Segment was drawn.

LET ABC be the segment of a circle, bisect the chord AC in D , and let DB be drawn perpendicular to AC , then join AB ; now make the angle BAE equal to the angle DBA ; and the line BD being prolonged, will cut the line AE at the point whence the segment was drawn, and from which the circle may be completed at pleasure.

APPLICATION.

Two instances are given; one where the segment is less, the other where it is more than a semicircle. The same directions apply to both. See Fig. 2.

All that has been said of the foregoing problem, applies equally to this; and, in the advanced parts of the science, such as in trigonometry, this problem will be found in constant use. It is highly useful in ornamental, as well as in architectural drawings, where, often we are compelled to resort to a center, before we can imitate small segments with truth.

The instance given in Fig. 3, will show the absolute necessity of ascertaining the center of one of the segments, in order to depict the whole imitation, according to the original. See also Fig. 5 of Problem LVII.

The painter, the jeweller, and others have frequently to unravel work far more intricate than the little specimen of

segments in Fig. 4; and as they cannot proceed by guess, it is plain they must resort to this problem for the means of executing what they undertake in this respect. The architectural draughtsman, who would faithfully delineate the various segments to be found in buildings, whether in the groining of arches, circular pediments, projections of mouldings, &c. must define their several centers, else he will not only lose their real proportions, but, if he has two or more to draw, he will never make them alike.

PROBLEM XVIII.

To cut a given Circumference into Two equal Parts.

LET the given circumference be ADB, join AB, and bisect it in C; describe a perpendicular from the point C, and it will cut the circumference into two equal parts at D.

APPLICATION.

By the solution of this problem, we have not only to cut a given circumference into two equal parts, but have to divide all the regular appendages thereto. This is highly useful in the section or profile of a building, and suffices on all occasions where only a bisection is required, without a view to ascertain the center of the circumference. The same means will answer for the bisection of any regular figure; and, in the case of pyramids, parallelograms, &c. the apex, or summit being found, a line drawn from the center of the base thereto, whether the figure be inclined or upright, will divide it into two equal portions, as in Figures 2, 3, 4, 5.

Such circumferences as stand even on their bases, as Fig. 4, will be divided by bisecting their bases at right

angles; but where they oblique to a point, as in Fig. 5, the angle must be divided; and in inclined figures, or where the upper parts do not measure equally with the bases, both the upper and lower lines must be bisected, as in Fig. 2 and 3.

This rule holds good in every instance where the bases and summits are under the same parallel; but in an inclined cone, the case will be different, unless the measurement be taken to the center of the base, for the figure will be an irregular parabola, which cannot be divided in this manner. See Fig. 6.

In the last figure it is obvious the horizontal line A B takes off more from one side than from the other; consequently the division made by the line C D must be unequal, for A C D must be considerably larger than D E B. This distinction should be held in mind.

Remark, that the term circumference properly applies to the circle; but I have used it on this occasion, as being best suited to explain the outlines of figures to be divided.

PROBLEM XIX.

Two Right Lines being given, to find a Third proportional to them.

THIS operation may be done in two ways, namely, so as to produce a line shorter than either of the given lines, or to produce a line longer than the greatest of them; but, in either case, the whole are to be proportionate.

Make any angle at pleasure (not exceeding 90°) with the two lines given, placing the longest of them, A B, for the base; on the produced line, A F, measure off A C, equal to A R; produce the line A B, by adding thereto the length of A C, which will carry it on to the point D; draw the

line CB , and its parallel DE , which parallel being intersected by a continuation of the line AC at the point F , will give CF as a third line; which will be less than AC , in the same proportion as AC is less than AB . See Fig. 1.

And if it be required to produce a line exceeding the greater line AC , in the same proportion that AC exceeds AB , the operation must be performed differently; for in such a case, the shortest line, AB , (Fig. 2) is to be the base, and AC is to be added thereto; then drawing the line BC , and its parallel, DF , the continuation of the line AC will, as in the first instance, intersect the parallel DF , giving the third line, CF , which will be greater than AC , in the same proportion that AC is greater than AB .

And thus any scale of proportion may be adopted, and continued at pleasure; for, if we make the difference of a tenth, an eighth, a seventh, &c. in the length of the two first lines, AB and AC , the third line will possess a proportional increase, or diminution of length, as it may be treated, whether according to the first or second process: we have but to omit the longest line, AC , (Fig. 2.) if we want a fourth shorter line; or to reject the shortest line, AB , if we want a fourth longer line, taking the remaining two for the operation, and the line required will be readily found. Thus any succession of proportionals, either increasing or diminishing, may be obtained.

The proof of lines being proportional is, that when placed at proportional distances, however near or remote, the inner lines will be touched at their points, by two lines joining the ends of the two outermost.

The 5th figure under this problem, will give a full idea of the truth of this position; the arches of the bridge diminish in a regular ratio; that at the center being 20

feet span, the next falling off one-fifth, which reduces it to 40 feet span; the next falling off a fifth of that 40, which reduces it to 32 feet span; and the next would fall off a fifth of that 32, which reduces it to $25\frac{1}{5}$. All the piers are 10 feet in breadth. Now a line drawn from the summit of the largest arch to the summit of the smallest, will touch the summits of the two inner arches, because the diameters of all are regulated by a ratio equally affecting all. This in particular relates to bodies possessing diameter, which are in a regular state of diminution, as the arches in question; but we have also a proportion which relates to equal distances of lines not having diameter, by which we ascertain the length of a line, that, being placed equidistant between the two others, should be touched by a line passing from the point of the first to the point of the third; the whole resting on one base.

It is on this principle the organ-builder increases or diminishes the length and bore of his pipes, and that the piano-forte-maker calculates the length and thickness of the wires which are requisite to produce particular notes: an increase of size produces a correspondent and proportional increase of tone; that is to say, the more substance there may be, whether by means of additional length, or of additional thickness, the more deep or grave will be the tone or sound.

In perspective, the heights of objects are determined by their distance from the eye of the observer, as in Fig. 3, where the row of houses, AB, though each is absolutely of the same height as that at A, appears to dwindle into nothing, and to vanish altogether; therefore the point B is, in perspective, called *the vanishing-point*.

This enables the draughtsman, or architect, to ascertain with precision what is the actual height of any object, which will in perspective be reduced in size, according to

the distance at which it is placed; by bringing the object forward with its due augmentation, as it advances in the triangle CBA, by means of this problem, to his front line, or base, CD, he would be able to compare, indeed, to measure it by known altitudes. It should be understood, that, in the science of perspective, diminution gives distance; and, that objects of the same magnitude, as the houses in the row CB, appear to diminish in proportion as they recede from the eye of the observer.

In the same manner two rows of trees, though planted perfectly parallel, will appear to a person standing at E, that is, in the middle between those next the front, CD, (Fig. 4) to converge both vertically and horizontally; i. e. they will seem to vanish in the point B, though of equal heights, and at equal distances throughout. Place them on a drawing, at equal distances on the line CB, as on the right, and they will by no means represent equal distances in perspective; but if the situations of the first and second trees, be once obtained, taking their heights as two given lines, the same as we have shewn regarding the lines AB and AC, in Fig. 1 and 2 of this problem, the third line produced by their operation, under the lessening principle of Fig. 1, will give the height of the third tree, which, with the height of the second, will give a fourth, and so on to the end, or, at least, so far as could be distinguished. When the height is found, and drawn parallel to the first tree, its place in the triangle, GBD, cannot be mistaken; for there can only be one spot where it can stand parallel to GD, and have its two ends touched by the lines GB and DB.

It matters not how acute or obtuse the triangle DBG may be, the proportions will still be the same, and the base line BD, as well as the upper line BG, will both be cut by the proportionals thus obtained, into proportional

parts, each respectively; that is, the trees will stand at such distances as will divide those lines into regular decreasing ratios, as on the left side of Fig. 4.

The fact is fully exhibited in Fig. 5, where we see, that while the arches diminish by a regular gradation of one-fifth, deducted from the span of the next greater arch, their respective altitudes also diminish in so regular a manner, that, as before specified, a line drawn from the summit of the greatest to the summit of the smallest, touches those that are intermediate. We cannot but admire the happy effect produced by such symmetry.

We have, however, other scales of proportion, viz. arithmetical and geometrical; the former increases always by the addition of a fixed number, as 2, 4, 6, 8, 10, in which 2 is the difference between any contiguous numbers: this is the proportion in which we estimate the ascent of stairs; for the height of one step being known, we at once decide upon the height of the whole, by adding as many such heights as there may be steps; or, in other words, by multiplying their individual height by their number.

The other, *i. e.* geometrical proportion, augments by always doubling the last quantity, as 3, 6, 12, 24, 48. The ends of parallel lines standing on the same base, and increasing according to this proportion, if placed at equal distances, generate the segment of a circle.

PROBLEM XX.

To find a mean Proportional between Two given Lines.

LET AB and BC be the given lines; put them together in a right line, bisect their united lengths in E, from

which point, as a center, draw the semi-circle ADC , and draw BD at right angles to AC , so as to fall on the point B , then DB will be the proportional required.

APPLICATION.

As the foregoing problem, (No. XIX.) generally speaking, applies more to the finding a third, which shall exceed the greatest of the given lines, in an equal ratio, as that greatest exceeds the lesser, so this problem more particularly relates to the finding an intermediate, or mean proportion between two given lines.

The line DB possesses the double property of standing as a mean between the two lines AB and BC , if placed parallel, and equidistant from each, and of being in itself the foundation of a square whose contents measure precisely the same as the area of a parallelogram formed by those two lines. This, it is to be remarked, approaches closely to arithmetical proposition, inasmuch as it differs from the greater line AB , exactly as much as it does from BC ; for if AB measures five feet, and BC measures three feet, then DB will measure four feet; that is, if the amount of the greater and lesser lines be added together, the middle line will be half their joint amount.

Arithmetical calculations are often so burthened with fractions, especially where great accuracy is required, that it becomes next to impossible to set off on paper those minute proportions which figures no doubt can furnish; hence we sometimes find it convenient to have recourse to mathematical operations, in which, by having lines given instead of figures, we arrive at the same information by brief and easy means. We might, for instance, feel extreme difficulty in ascertaining numerically the extent of a line whose square should be commensurate with a parallelogram, whereof the long sides

should be $24\frac{1}{2}$, and whose short sides should be $16\frac{1}{4}$, whereas with compasses and scale we should not only calculate, but actually produce a line appertaining to the square in question. Sometimes a scale is not at hand, yet this problem may be solved by taking a pencil for a director, as to correctness of line, and a pair of compasses for laying down the lengths of lines, for drawing the circumference, and the perpendicular, and for measuring the product.

PROBLEM XXI.

To cut a Right Line, so that a Rectangle, (or Parallelogram) having the whole Line for its Length, and One Segment for its Depth, shall be equal to the Square of the other Segment.

Let AB be the given line, describe its square BCD ; bisect AC in E , and draw the diagonal, or oblique line EB ; produce EA to F , so that it be equal to EB ; describe the square $AFGH$; and the line GH continued to K , will cut off a parallelogram, $HBKD$, equal to the square $AFGH$.

The line AB is by this process divided into extreme and mean proportion, in the point H ; that is to say, the whole line bears the same proportion to the larger segment that the larger segment does to the smaller.

APPLICATION.

This, as well as the preceding, is an inversion or change of figure of Problem XIX; it relates particularly to the system of commutation, of which several instances will be hereafter found.

PROBLEM XXII.

Triangles of equal Base and Altitude are equal to each other.

THE contents of a triangle are ascertained by its height being multiplied by half the base; and this is always the case, whether the vertex or summit of the triangle stands over, or projects beyond the base.

Take three pieces of paper, and form the three triangles ACB, BDE, EFG; cut the first into two equal parts, from C to 1, and one half being reversed, will fit the other, and form a parallelogram. Cut the second in the same manner, at the figure 2 to 4, turn it up so that E shall join D, and another parallelogram exactly similar and equal to the first will be constructed. The third triangle is thrown far out of its line, so as to cover the extent of twice its base; cut it from G to the figure 3, and carrying the point 3 to 5, also the other portion, so that F may come to the place of E, and W to Z, then there will be seen a horizontal parallelogram equal to those resulting from the first and second triangles.

N. B. The altitude of a triangle, and, indeed, of all figures, is measured by a perpendicular from its vertex, or summit, to the line of its base, as from F to H in the last figure. In the first the dotted line C 1, and in the second figure, the perpendicular, D B, are the altitudes.

APPLICATION.

This is a most important problem, inasmuch as it relates to surveying, and to architectural purposes. It is the basis of an infinity of operations, and teaches as a

truth what is not sufficiently obvious to the eye; viz. that, however much extended, or oblique a triangle may be, yet, if it lay between a certain parallel, and have the same extent of base with another which may be more compact, and appear longer; yet, that, the area, or surface, of each will be exactly the same.

I recollect seeing a gentleman greatly embarrassed for want of knowing whether he should save materials by finishing his house with a pavilion roof, or with gables. The carpenter he employed insisted on the pavilion being the cheapest as to materials; whilst the mason gave a firm opinion that the gables would save much wood and slate. The fact was, that the carpenter knew the pavilion was most expensive, while the mason was intent on the additional quantity of work he would be paid for, if the gables were run up: neither of them, however, understood that the surface to be covered, being under the same angles, must be alike in either mode; and, that, consequently, the amount taken off at the upper corners of the front and back of the pavilion-roof, would exactly complete its two ends, as may be seen by the following sketch. See Fig. 2.

Let the roof be made, so that from A to C be the ridge, and E G, F H, be the eaves of the front and back; the distances from C to D, and from B to A being less than from G to D, or from B to E; invert the triangle B A E, changing the side B E into the place of the side B A. When the roof is raised to its proper angle, as may be seen, if a model be cut out in paper, the triangle B A C will fit exactly, so as to join in at the sloped line E A, and will cover in half of the end. The other three corners being done in the same way, the whole of the roof will be completed, without either excess of, or any demand for, more materials. But it should be observed, that the four-corner timbers, F A, E A, H C, and C G,

must be longer and stouter by about one-third ; and, that, in such case, much room is lost. For this reason we see barns, lofts, &c. where space is an object, always built with gables, which, indeed, have the further advantage of not being so subject to leak at the corners. Certainly, there is an additional expense for the masonry ; but, where chimnies are to be run up at the ends of a house, that makes a small difference, as may be seen in Fig. 3. Those who build, must be guided by the comparative expence of wood-work, and of masonry.

In saying that the areas of triangles, under the same parallels, and standing on equal bases, are the same, I must not be understood as describing their circumferences also as being equal : on the contrary, the more compact the triangle may be, the shorter will be the line by which it is surrounded ; and the more it may incline, or hang over beyond its base, the longer will that line of circumference be. For, in some instances, a very small augmentation adds considerably to the area ; while, in others, it effects comparatively nothing ! There is an old puzzle which has, to my knowledge, perplexed many very clever men ; it is apt to my present purpose.

A shepherd had one hundred hurdles for folding his flock, and on being told by his neighbour that they were not placed to the best advantage, the shepherd commonly fixing them in a long parallelogram across the field, wagered that position made no difference as to the space they inclosed. The neighbour insisted, that, by means of two hurdles more, he would double the area ; and he gained his wager : for as the shepherd had placed only one hurdle at each end, and forty-nine at each side, as in Fig. 4, it was evident that by doubling the length of the two ends, so that each should have two hurdles, as at 2, the area would be twice as large as before ! But if those two additional hurdles had been placed in the two lines

of forty-nine each, as expressed by the dots at 1, it is clear they would only have added one forty-ninth part to the extent of the inclosure.

And in Fig. 5, we see that the same *perimeter*, or line of boundary, may inclose two figures, in form of stars, the one of whose contents shall be equal to eight or ten times the space contained within the other.

But whether the triangle 1, 2, or 3, in Fig. 1, be found on a roof which is to be covered with lead, the quantity of materials will be precisely the same, while the work will be infallibly increased by the quantity of out-line which is to be cut, soldered, &c.

PROBLEM XXIII.

Parallelograms of equal Base and Altitude are equal to each other.

EVERY square, rhombus, rhomboides, and parallelogram, may be divided into two equal and similar triangles, by a line drawn obliquely to the opposite corners; therefore as, according to the foregoing problem, all triangles of equal base and altitude are equal to each other, so must all parallelograms standing on equal bases, and under the same parallels, be equal in contents.

Cut off AED, by means of the perpendicular ED, and it will be found to complete FGB; in like manner bisect the 3d figure in HI; and, as in the case of triangles, bring I up to F, and K to H, to complete the parallelogram.

APPLICATION.

What has been said of the triangle, relates in every point to the parallelogram; and, it must be obvious to the

builder, the 'plaisterer, and the plumber, that whether the surface be at right angles, or inclined, the same quantity of masonry, plaister, or lead, would be requisite. If a flight of steps is required, though the tread, or flat of each must extend in proportion to the slant of the figure, yet the same height would be needful for each. By this we learn, that the more upright the stairs are, the less materials will be required for gaining the same height. Considering the figure given as horizontal, we shall find that a field of either form would contain as much crop as the others, while the quantity of hedge, &c. to inclose the oblique field, would be increased in proportion to that obliquity. This may seem paradoxical, but if we suppose a number of fields to lay contiguous, all greatly inclined, as in Fig. 2, they would occupy the same space as an equal number, Fig. 3, standing at right angles, under the same parallels, and of equal bases; while the latter would assuredly have less hedge-row, and be inclosed at a much smaller expence.

This shews how absurdly those persons act who are not attentive to rectangular disposition in laying out lands, &c.; by which neglect, not only facility of cultivation is impaired, but an actual increase of expence is incurred!

Perhaps this may be the fittest place to remark on the disadvantages, in some respects, of lands that lay on a slope. Many persons purchase what are called hanging, or side-long lands; measuring them by the chain. If such be appropriated for pasture, or for crops which diverge and become a close covering over the soil, little objection can be made; but, in the case of application to building, or where a succession of perpendiculars, either obvious, or occult, is intended, the loss of space will increase as the land becomes more inclined, *i. e.* as it deviates from the horizontal. For, it is clear, that an acre measured on an angle of 40° , will produce only $\frac{3}{4}$ of an

acre in buildings, as may be seen in the annexed Fig. 4; where the slant line, AB , or hypotenuse, is one-fourth longer than the base AD , which base would be all the level the acre of inclined land would produce.

Assuredly there may be an acre of grass on the side AB , but the base AD would allow only three roods of edifice; nor could we expect more timber to grow on the slant than could stand to grow on the base; for timbers are implied perpendiculars, or vertical lines, of which no more could fall on AB than could find space on AD !

PROBLEM XXIV.

To make on a given Angle a Parallelogram, equal to a given Triangle.

IF the two foregoing problems be fully understood; this operation will appear remarkably simple. Bisect the base AC of the triangle ABC in D , draw BF parallel to the base AC , and from the point of direction, D , make an angle to H , similar to the given angle X ; draw also the parallel CF , and the parallelogram, $DHCF$, will be equal to the triangle ACB ; for a triangle of equal base and altitude with a parallelogram, is equal to half that parallelogram.

APPLICATION.

We now enter on problems which treat of commutation; a branch of the science that is of most essential service to all classes of artificers, &c. By them we learn to change one figure into another, of equal measurement. This problem alters a given triangle into a parallelogram, which is, perhaps, one of the easiest changes. For instance, let us suppose that a man has a piece of sheet

lead of a triangular form, which, not being suited to his purpose, he would change with a plumber for some of equal thickness, but according to a parallelogram. Now, the plumber might cut a deal of lead to waste, injuring either one or other party, were he to guess at the quantity he should take out of a large sheet; but, possessed of the information contained in this problem, he would, either at right angles, or according to any given angle, measure off, with sufficient precision, such a portion as should correspond in weight with the triangular piece to be exchanged.

The triangle, as I before stated, is admirably calculated for a medium by which to find the contents or measurements of other figures. Let it be required to estimate what sized tunnel would carry off the quantity of water contained in a stream, whose breadth should be nine feet at the surface AB, lessening gradually from the banks towards the center, CD, where it should be rather flat for about three feet, but inclined more to one bank than to the other. Fig. 2.

The depth, say, is five feet; now, as we have a parallelogram in the center, contained in EFCD, and that AEC and BFD are triangles under the same parallel, though of different bases, these bases joined, by taking out the center column, or parallelogram, would form one triangle, which being, according to this problem, commuted to a parallelogram also, would, when added to EFCD, form a rectangular figure of equal measurement with the above section of the stream.

It is to be observed, that this commutation is equally a subject for the next problem, the figure being right-lined, or so nearly right-lined, as to render the small difference of no consideration.

Problem XXVI. shews us how to change any parallelogram into a square, and as that furnishes us with the

diameter of a circle whose area is equal to that of the square, we cannot be at a loss how to estimate the size of a conduit, or of a tunnel, capable of carrying off the whole body of the stream in question, as thus: the section of the stream, which is a trapezium, would form a parallelogram, which parallelogram would form a square.

Now, as the diameter of a circle is to that of a square of equal contents, as 10 to $8\frac{7}{16}$, the circle would be of the same area as either the section, or the parallelogram, or the square. Thus, by progressive means, we are enabled to change to any form that may be necessary.

PROBLEM XXV.

To make a Parallelogram equal to a given Right-lined Figure in a given Angle.

LET $ABCD$ be the given right lined figure, and E the angle given; now, as the figure given is a trapezium, and, consequently irregular in all its sides and angles, it is necessary to bisect it by the diagonal DB , by which means two triangles of equal base will be found; this diagonal gives the height for the parallelogram $FHGK$; its breadth is determined by a line drawn, A , perpendicular to DB ; the half of which furnishes the breadth of the parallelogram, $LGMK$; the other triangle, DCB , being transformed in a similar manner, into $GFHK$, and placed beside the line GH , completes $FLKM$, equal to the figure given.

N. B. Attention must, of course, be paid to the given angle, E , which must be imitated in forming the parallelogram.

APPLICATION.

All right-lined quadrilateral (or four-sided) figures may be divided into two triangles, each of which must have

one side corresponding with one side of the other, or, what is usually termed, "common to both." This equality serves for the altitude of a parallelogram, whose base will be found by the semi-diameters of the two triangles, which, when conjoined, will form a regular figure of equal contents with the given right-lined figure.

This commutation is extremely simple, and, like the preceding, applies to many useful purposes. For instance, if a road should be carried through a person's grounds, and that he should be entitled to an adequate portion of land from an adjoining common, he would be enabled not only to compute the exact quantity, but to apply it to the angle his own land might make; so as to render the whole compact. Problems XXVII. and XXVIII. would come into use if it were requisite to make the new acquirement uniform with any other plot; as will be seen when those problems come under consideration.

Again, suppose a slater should wish to know how far the slates on a certain roof would apply towards covering another building, which is a most important point, when purchasing old materials, &c.; he could not be at a loss regarding the extent, and, consequently, of the value of the slate. We often see men puzzled by an irregular form, which, to the novice in mathematics, presents great embarrassments; but, when the facility of commutation is understood, these difficulties vanish, and he who before made his purchases or calculations at random, assumes due confidence, and is qualified to enter on those speculations, which, till then, must be precarious and dangerous. Men of information are always seen to stop at a safe point, beyond which ignorant persons proceed, and are disappointed, at least, if not ruined.

It is precisely the same with him who undertakes flooring, plaistering, digging ground, or any other job; if the object be diagonal, and unequal in the length of its sides, few can estimate correctly the real extent of the undertaking: they ordinarily measure along and across the greatest breadths and lengths, but in this mode many great errors are committed.

In those gross concerns, where the materials or the labour are not of great value, the damage may not be enormous; but where a material is of high price, and the labour proportionate, much injury may follow. I will state a case: let it be required to make a rich carpet for a room of an irregular form, as in Fig. 2, and that the maker should measure along the greatest depth, and the middle of the length; the result would prove far wide of the fact, as to real quantity; for the lines AB and CD , drawn in the above manner each way, would cause the demand to be under what it should be, by near a fourth part! Here the maker would be the loser, but if he had to buy, he would be equally a gainer.

PROBLEM XXVI.

To make a Square equal to a given Right-lined Figure.

LET A be a given right-lined figure; make the right-angled parallelogram, $BCDE$, according to the last problem; then produce one of the longest sides, say BE , to F , so that EF be equal to ED ; this done, bisect BF in G ; from which, as a center, describe the semicircle BHF ; continue DE to H , and the line HE will be the side of a square equal in area or contents both to the original figure, A , and to the parallelogram $BCDE$.

APPLICATION.

The square cannot be immediately formed from any irregular figure; but by the medium of the parallelogram, as exhibited in the last problem, that object is readily attained. The learner will observe, that Problem XX. was given as preparatory to this, which, in a very easy and concise manner, produces a square of equal area with the given right-lined figure. Problem XXI. taught how to cut off that proportion from a line which should be the depth of a parallelogram, having the whole line for its base, and which parallelogram should be equal to the area of the longer segment of the line so divided. This, in a manner, inverts that problem, by shewing how a square is discovered which shall be equal to a parallelogram, derived from, and equal to any given right-lined quadrilateral figure.

By aid of this problem, we are enabled to commute in various manners: it has already been seen, that a triangle can be converted into a parallelogram, and that all right-lined figures may be divided into various triangles, all of which may, by Problem XXVII. be brought to have equal bases, and to change into parallelograms during their application to that given line or base; at the same time, that they should partake of an inclination, or stand at an angle, common to all of them.

But, in this place, let the object be confined to the given quadrilateral (*i.e.* four-sided) right-lined figure; and suppose it to be a measure, say a bushel for corn, &c. which being of an inconvenient form, must be changed into a square, for the purpose of packing; we will further suppose that a half bushel, and a quarter bushel, are wanted, which should fit, the smaller into the greater. The given figure, A, being changed into a parallelogram,

and that being again commuted into a square, gives the surface of a measure whose depth must be equal to that of the original given figure.

It is obvious that half such a square will give half the measure; but, admitting that the sides were ever so thin, the half bushel of this form never could be put into the whole bushel; consequently its surface, which is a parallelogram, must again be converted to a square; as to the peck measure, that is easily made, by taking $\frac{1}{4}$ of the original figure, which consequently must be equal to a quarter of a bushel; and this holds good equally as to weight, for if any solid be of equal depth throughout, or that its depth be of such a regular form, say a triangle or wedge, as may admit of reduction to a regular figure, the plumber, &c. would be enabled to commute the irregular surface into a square, of which the bulk, and consequently the weight, should correspond with the given solid.

It very frequently occurs that we have room for goods in one form of arrangement, when they could not by any other means be retained. Thus wine-pipes set on their ends, as we see in the shops of liquor-venders, do not incommode so much as when placed horizontally; for we can oftener spare height than surface. A wine-merchant desirous of contracting the space on which his bins stand, could not, generally speaking, object to height, if length and breadth could be diminished. This is very easily effected, without changing the admeasurement of the division internally. Bins are often made low, with oblong fronts, and project as oblongs from the walls. To remedy this inconvenience, which often is of some moment, he should commute the horizontal oblong, which forms the surface of the bin, into a square by this problem; this will shorten the depth from the wall considerably, but cause an increase of breadth; but this is

only during calculation, for when he proceeds to commuting the front oblong into a square, the extension will be reduced, perhaps, with some gain of space, so that both the length and the breadth would be abridged. As the difference must be made up somewhere, we must conclude the height of each binn must be increased, and such would be the case.

The foregoing is given as an illustration of the very easy means whereby many such matters may be effected, and to shew the utility of knowing how to commute from one form to another. The problem applies to an infinity of circumstances, relating to almost every branch of business!

I should here remark, that the square may be reduced to a triangle, by drawing a diagonal line from two opposite points, A B, extending the base to double its former length, as from C to D, the triangle A B D being double as long, and of the same altitude as the square, must be equal to the square. See Fig. 2.

The summit of the triangle may be any where between E and B, or indeed any where under the same parallel of altitude; as already proved by Problem XXII.

If it be required to change a square into a parallelogram of which one side is given, let the side of the square, B C, Fig. 3, be placed perpendicular at the end of the given side of the parallelogram A C; then draw the line A B, which bisect in E, and the perpendicular drawn from E will cut the line A C in F, which will be the center of a semicircle, whose diameter will give the proportions of the parallelogram sought, for A C will be the length, and C D will be the depth of a figure equal to the square of B C; and it will be the same, whether the longer or the shorter side of the intended parallelogram be given: the other side cannot fail to appear; for the perpen-

dicular drawn from H, where the line BD is bisected, equally points to F.

Let me ask, how could a printer better ascertain, to a nicety, what sized page of oblong octavo would be requisite to contain the quantity of similar letter-press in the page of quarto? I mean mathematically; for, as to reference to a set of tables, that is out of the question: take away his book of knowledge, and, like a soldier without his arms, he is no better than any other man unskilled in the profession.

PROBLEM XXVII.

To apply a Parallelogram to a given Line equal to a given Triangle in a Right-lined Angle.

LET the line given be AB, the given triangle C, and the given angle D; in an angle equal to D, make the parallelogram BEFG equal to the triangle C; let BE be placed on a line with BA, and produce FG to H, drawing the parallel AH, and joining HB; continue the line FE to K, until it is cut by the continuation of the line HB; then from K, draw the line KL, let GB be continued to M, and HA to L, thus will the parallelogram BMAL be equal to the parallelogram BEFG, and to the given triangle C.

APPLICATION.

Here we have the method of commuting, and of applying also, a triangle as a parallelogram to a given line, and at a given angle. The various uses of commutation have been already treated of, but it remains to say of this,

that, what on most occasions requires two operations, is here effected by one. This is particularly useful where various figures of different form are to be computed under one. Suppose, for instance, that two persons possessing land should have their several properties so blended, as (Fig. 2) to produce an amicable determination to exchange measure for measure, in order that each might have his own, collected as it were, within a ring-fence, as in Fig. 3.

Triangular fields are not uncommon; but, at all events, there are so many trapezia to be found among inclosures, that often, for the sake of brevity, triangular computation will be adopted. This problem particularly points out the mode of reducing that figure to a parallelogram, whose form should be regulated by an adjoining line. Thus a number of various figures may be appended to each other, all of one breadth, but their thicknesses varying according to their respective dimensions. The foregoing Problems, XXVI. and XXVII. come strongly in aid of this practice, for by them we see that the parallelogram may be changed to a square, or the square to a given parallelogram. To elucidate this more fully, I refer the learner to the example contained in Fig. 2, whose various figures are, in Fig. 3, collected upon one base line, like so many layers, each answering in contents to the corresponding marks in, and abstracted from Fig. 2; so as to leave the residue thereof one uninterrupted property.

The numerals opposite to each figure in N° 2, refer to the problem, by which the change is effected; observe, that, B having five sides, the triangle cut off by the dots is calculated separately from the trapezium, though included with it in the corresponding parallelogram at Fig. 3.

PROBLEM XXVIII.

To describe a Right-lined Figure, similar to a given Right-lined Figure, and equal to another given Right-lined Figure.

LET ABC be a triangle to be brought to an equality with D , but to retain its original form. Make on the side BC a parallelogram, $BCLN$, equal to the triangle ABC ; change the figure D into the parallelogram $CFNM$ on the line CF , and on the given line CN ; then, by Problem XX. find a mean proportion between the lines BC and BF , which will be the base of a triangle GKH , similar to ABC : the sides are to be found by measuring the angle at G equal to that at B , and the angle at H equal to that at C .

APPLICATION.

A person gives to a jeweller several pieces of gold; viz. one large triangular piece, A ; three square pieces, of a size smaller, as B ; six oblong pieces of a third size, as C ; and twelve yet smaller, as D , all of one thickness, with directions to commute them into triangles, and to place them as they may fit, in two circles around the longest, which is to stand for a center. Now, as the whole of the pieces are to retain their original thickness, the jeweller would only have to ascertain, which he may do by this problem, what are to be the sizes of the triangles in each row, and thereby to regulate his proceedings. It is plain that he must reform all

the pieces, except the middle one, which is to serve as a standard for the rest.

The center-piece is an equilateral triangle, A.

The next three are square, and of this size, B.

The next six are oblong, and of this size, C.

The next twelve are oblong, and of this size, D.

The jewel formed of these, would appear in the proportions exhibited in the figure.

In this manner reservoirs of water, magazines for stores, military works, &c. &c. may be constructed on any scale, and according to any given figure. Weights and measures may likewise be regulated, or commuted on the same principle.

PROBLEM XXIX.

In a given Circle, to describe a Triangle similar to a given Triangle.

LET DEF be the given triangle; draw the right line GAH, touching the circle in the point A, at which any particular part of the triangle is to fall; say the angle D; draw also a similar line, ODK, across the angle at D; then make the angle HAC equal to DFE, and the angle GAB equal to DEF, draw the line BC, and the triangle ABC will be similar to the triangle DEF.

APPLICATION.

This, and some of the following problems, relate to the adaptation of one figure to another, either by being enveloped, or by enveloping the other. In the first instance, where the figure is to be contained in another, it is said to be *inscribed*, and, where it is to surround another,

it is said to be *described*: in either case, the angles of the inscribed figure are understood as touching the describing or surrounding figure, else the term is misapplied.

With respect to this branch of the mathematics, it must be premised, that, what applies to one problem, affects all; for it is obvious, that whether the object be to *inscribe* or to *describe* a figure with another, or whether the figures be triangular, rectangular, or circular, the fact and its utility amount to much the same; therefore, in lieu of burthening the work with useless repetitions, I shall briefly point out to the learner, that in every profession we are often expected or necessitated to include work within given spaces, and, in so doing, to proceed by similarity. Thus, if a plan be given, where the process has been arranged by a scale of one-tenth of an inch to a foot, when the design is to be executed, we must have reference to similar angles contained in similar figures by which means we cannot fail of imitating on the large scale, what was drawn as a guide on the small scale.

As I have already remarked, we are not to expect that a case of good instruments, or a chest of good tools, are ever at command; nor can we, like the artist in his chamber, determine the places of all the principal points, by frames intersected by numerous threads at right angles, as will be found under the head of enlarging or diminishing a plan, by which both the original and the copy are divided into an equal number of squares, so as to carry a just proportion throughout; on the contrary, we must act with our scale and compasses, else we should never be able to ascertain the manner in which one figure is described within the other.

The triangle given herein, divides the circle into three portions or segments, each subtended by a side of the triangle, which is properly the chord of that segment.

Suppose an ironmonger or a brazier should be making

a tripod, having a circular rim, he must first divide that rim into three equal portions, when the places for affixing the legs or stand to the rim would be given; but such could only result from an equal division; for, if the triangle be irregular, the workman must determine, by mathematical research, where its points should rest. This problem would be the basis of his operations for inscribing, while the next would be the guide for describing.

In ornamental, and, indeed, in various useful works, we have to inscribe work within a circle; and, although such work may not be triangular, yet, for the sake of accuracy, we consider some of the most prominent parts as being points of triangles, or of other figures; observing, that where such parts may not exactly coincide with a circle drawn around them, we can continue their direction until the circle be touched, as may be seen in Fig. 2, where A and B, which touch the circle, give two points of the triangle, and the third point is assumed by continuing C in its proper direction: the place of the head, and of the toe, are further ascertained by dividing the triangle, as in Fig. 2. Were the pattern of the same dimensions with the work, this would be a tedious way of doing what might be so easily effected by measurement from point to point; but the variation in size compels us to resort to this problem for aid.

Fig. 3 is a fan-light, in which are circles that have triangles inscribed, and triangles in which circles are inscribed: the latter are the subject of Problem XXX.

PROBLEM XXX.

To describe a given Triangle round a Circle.

LET ABC be the circle, and DEF be the given triangle, to make a triangle equiangular therewith; produce

the side EF to the points G and H , and find the center of the circle K , and draw the perpendicular KB ; then make the angle BKC equal to DEH , and BKA equal to DFG . The lines LN , MN , and NL being drawn touching the circle at right angles, with the three rays proceeding from K , will form the triangle around the circle, equiangular with the given triangle.

APPLICATION.

What has been said of the foregoing problem, applies equally to this.

PROBLEM XXXI.

To inscribe a Circle in a given Triangle.

LET ABC be the triangle given, cut the angles ABC and BCA each into two equal parts, by two right lines meeting in D , which will be the center of the intended circle; draw also the three rays, EF , and G , as in Problem XXX. by which the accuracy of the work will be proved.

APPLICATION.

It sometimes happens that corners are cut off from apartments, for the purposes of carrying up chimnies, or for small back stairs; in the latter case the center of the triangle must be ascertained by this problem, when the requisite length of each step will be known, and the ascent will be regulated by a spiral revolution around that center. This, for the most part, takes place in rectangular triangles, of which the hypotenuse makes a fifth side at the corner of the apartment, into which the door, A , leading to the stairs, opens, and offers the opportunity of gaining a few steps in an oblique direction from the

center, as in Fig. 2; also of making a recess, or closet, B, under the steps, as they rise towards the close of the first revolution.

Triangular pediments to large mansions, as Fig. 3, or to turrets for clocks, Fig. 4, are very common; in the former circular windows are generally placed, which, though they may not be so large as to touch the corners of the pediment, must nevertheless be centrically situated.

PROBLEM XXXII.

To describe a Circle about a given Triangle.

LET ABC be a given triangle, bisect the line AB in D, and the side AC in E, and from the points D and E draw lines at right angles to those sides; they will meet in F, which is the center, whence a circle being drawn, will touch the points A B and C of the triangle given.

APPLICATION.

Suppose it were required to know what space would be occupied by a body of a triangular form revolving on the axis F; it would be necessary to ascertain the center, and thence to describe a circle; for the triangle may be very unequal, that is, all its sides may be of unequal lengths. In the case of a triangular boring-tool, little difficulty could arise, since such implements are, in general, made equilateral, and the center is very easily known; the space occupied by its revolution would be all around to the same extent, as from the center to any one of its points.

This problem likewise affects ornamental subjects, where, as in Problem XXIX. Fig. 3, it might be required

to draw a circle around a given triangle; or where, as in Fig. 2, the object to be surrounded might offer three points, by which the circle should be guided, and touch them all.

PROBLEM XXXIII.

To describe a Square in a given Circle.

DRAW through the center of the circle two diameters, cutting each other at right angles in A, join their several ends, where they touch the circle, by four lines, from B to C, from C to D, from D to E, and from E to B, and the square will be formed.

APPLICATION.

This problem is in constant use with immense numbers of mechanics, who rarely understand its principles, but conceive they are squaring a circle, a matter of such difficulty, that no mathematician yet known has ventured to pronounce effected, or to offer any rule whereby it may be done with absolute precision! It is true, we are so near the mark, as to leave only the few who aspire to the most sublime parts of the science, to regret that so trifling an inaccuracy should remain.

The timber-merchant, and the several workmen employed either in trimming, sawing, or valuing whole trees, all compute by the square, which determines the number of solid feet in a tree; consequently, they must be in the habit of squaring timbers, and of taking their dimensions, both from the ends, and from the girth, or circumference in the middle, and other intermediate parts; but, it is to be carried in mind, that on many occasions, a large portion of the wood is cast out from the

work, and turns to little value; thus, if a round tree, at its smallest end, included a square of fifteen inches, and at the butt, or lower end, included a square of twenty inches, and that the timber is to be of the same thickness throughout, all the extra bulk of the butt will be carried away in the slabs or side-pieces taken off by the sawyer.

The first object, on such occasions, is to see that the small end be cut perfectly at right angles, *i. e.* perpendicular to the length of the tree, for any obliquity will necessarily diminish the diameter one way, as may be seen by the example in Fig. 2, where the square, or perfect diameter, *AB*, is of the same length with the obliqued diameter, *CD*, which latter would occasion a deficiency of timber, if followed as a guide for trimming the tree to a square.

The square being marked at one end, and a line drawn so as to take off a side slab, the whole length, at right angles, to the end so squared, the sawyer soon removes that slab, which then presents a level surface, and enables the director to draw two other side-lines, right and left, in the same manner, and perfectly parallel; these slabs being taken off, the fourth side is trimmed by means of a line drawn parallel to the first, which completes the square; but all must depend on the exactness of the original square, which being faulty, would cause the timber to be a rhombus parallelopipedon, that never could rest properly in its place, and which, if sawed into planks, would both cut up to disadvantage, and give great trouble to the sawyers, who would have to follow an oblique line, as may be seen in Fig. 3, which represents the end of a timber falsely squared.

If it were sawed in the lines *AB*, *CD*, which are not perpendicular to the surfaces *EDAF*, two large triangles, *ABE*, and *CDF*, would be made, and occasion great loss.

It has before been explained, that a square is to a circle, of equal contents, as $8\frac{7}{8}$ to 10; that is to say, a square of which the diameter is 8 feet $\frac{7}{8}$, will contain an equal space with a circle whose diameter is 10 feet; and it is on this calculation, the modern mode of measuring round timber is founded, which gives the full amount of solid contents, but includes a quantity which the person squaring the tree finds little use for, though paid for as timber. This will be seen by Fig. 4, which exhibits how a square fits upon a circle of equal contents; the four slabs, 1, 2, 3, 4, are shewn, and the four corners, 5, 6, 7, 8, included in the measurement, by which the seller gains an advantage of about two-thirds more than can be brought into the actual squaring of the timber, as contained in the inner square.

PROBLEM XXXIV.

To describe a Square about a given Circle.

DRAW two diameters, as in the preceding problem, at right angles, and at their terminations draw four lines at right angles thereto, viz. AB, BC, CD, and DA; they will form the square about the circle.

APPLICATION.

The frame-maker is often directed to make a square frame for a round picture, and the carpenter is not unfrequently employed in making square cases, or guards, for cylindrical objects, such as the leaden pipes descending from the tops of houses, which in many instances are secured from pressure and other injury by this precaution; the latter, assuredly, may be done pretty near the

mark by guess, but with a waste of material, and a want of due squaring: as to the former matter, exactness is required, and the circle in question must be treated according to this problem.

It would, indeed, be endless to point out the immense variety of instances in which this problem is indispensable; suffice it to say, that a very great portion of the operations in almost every handicraft business have, either directly, or indirectly, reference thereto.

PROBLEM XXXV.

To inscribe a Circle within a given Square.

BISECT the two sides, AC and BD of the square, and draw EF , bisect EF in G , which will be the center, whence a circle, drawn with the radius EG or FG , will touch the four sides of the square within.

APPLICATION.

Although, as before stated, figures described, or inscribed, ought, strictly speaking, to touch each other respectively, yet the various occasions in which circles are required to be placed centrically, in squares, necessarily admit of deviations from this rule, on the same principle as circular windows may be made in pediments of houses, where they are not by any means necessitated to touch the triangles in which they are inscribed. Thus, a tin ventilator, placed in a window, may be of any size, without reference to the size of the square or frame of glass in whose place it stands; but, to place the ventilator awry, would not prove creditable to the workman; he would, necessarily, seek the center of the space, whether square

or oblong, in which the ventilator should be placed, and having found the center of both the greater and lesser diameters, would there fix the center of the machine ; for that spot must, in a rectangular figure, be equidistant from each of the corners, and, consequently, must be its true center. See Fig. 2.

With regard to works of ornament, such an expanse of field for the exercise of this problem offers, as to render it unnecessary to discuss its merits, or general utility ; like the preceding problem, it regulates, either openly or occultly, at least half the work done in jewellery, chimney-ornaments, cieling compartments, mosaic inlaying, preparations of wood for turning, and a thousand branches of business requiring accuracy as to the division, or appearance, of the work.

• PROBLEM XXXVI.

To describe a Circle about a given Square.

IN the square, A, B, C, D, draw the diagonal lines AD, BC ; the point E, where they cross, will be the center of a circle, which, taking EA, or EB, or EC, or ED for a radius, will surround the square, and touch each of its corners externally.

APPLICATION.

This, like the preceding, is applicable to a multiplicity of occupations, either in whole, or in part ; for whether a segment of a circle be taken, or the entire circumference be in use, still this problem must be resorted to ; for instance, let it be required of a builder to draw, over the windows of a house, ornaments which should be qua-

drants of a circle; in the first place, he would, from the upper part of the window, unless it were a square one, set off in depth, AB , CD ; measuring equal to its breadth, AC , Fig. 2. The diagonals, AD , CB , would intersect each other at their common center, E , and the line AE would serve as the radius of a circle, of which the segment AGC would be a quadrant, or fourth part; and, if an angular ornament were required, the division of that segment into two equal parts, at the point G , would be the peak of a triangle, AGC , on which the formation of the ornament would be founded.

But it must not be supposed that these problems relate merely to circles and squares; they regulate the situations of various figures to be included, or enveloped in others; for instance, the coach-painter having to place a coat of arms, or crest, on the side of a carriage, would find the center of the pannel, and thus give his workmanship at least the air of regularity, whatever might be his abilities in his profession. Attention to determinate points is not only in itself necessary, but rarely fails to distinguish those by whom it is practised. A very indifferent artist in this, or in any similar branch where measurement is an essential, will often determine the spectator in his favour, merely by the exactness of locality, while a far more able hand shall be denied the reward of superior talent, because the eye were offended by some slovenly lapse, or by inattention to exactness of angle, or of situation.

Painters are frequently at a loss how to place their figures, so as to occupy a given space regularly; they are unable to distribute the various points into their proper situations, from want either of reflection, or of instruction; to exemplify this, let it be required of a herald-painter to place a crest, representing a hand and dagger,

in the proper part of a circle, and that circle in the proper part of a pannel; the center of the pannel will often be different from what at first sight should appear such; but the rule for ascertaining the perspective center is very easy. Let the shape, A, B, C, D, be as in Fig. 3. Bisect the chord BC in K, by the line AD, draw OK parallel to AB, then bisect AD in E, and through E draw EH, parallel to AO, the line EH will be the side of a square, E, H, G, K, from whose center, V, the circle should be drawn; now, the crest, viz. the hand and dagger, may be represented by a triangle, thus:

Let the base line be ZX, and the summit be S, Fig. 4 and 5, find the center of the triangle, which being brought to the center of the circle, the crest will be duly inscribed therein.

Thus any figure may be inscribed by a judicious arrangement of the triangle, taking care to select the most prominent points, or those farthest removed from the apparent center, for the angular points, so that the rest of the figure may all be comprehended within the circle, and balance as nearly as its form may admit. It is obvious that many figures will never balance in a circle, in which case it shews a want of taste and judgment, when an attempt is made to circumscribe them.

PROBLEM XXXVII.

To describe an Isosceles Triangle, having each of its Angles at the Base double to the Angle at the Vertex, or Point.

Cut any given line, AB, in the point C, so that the rectangle contained under AB, BC, be equal to the square of AC (Prob. XXI.); then about A, as a center,

with the distance AB , describe the arc BD , draw BD equal to AC , and join AD ; the triangle ABD will be the isosceles required.

APPLICATION.

This problem is preparatory to the division of the circle into regular polygons, and requires particular attention. The mechanic is often at a loss how to divide his circle into particular portions, and, perhaps, we have no mathematical operation more difficult, because we have no check whereby we can control, or detect growing error; the fault is not known until the concluding moment, when the whole must be recommenced under the same disadvantages, which, if any part of the instruments be faulty, or the hand or eye be inaccurate, will again produce the same disappointment.

The mode laid down for making an isosceles triangle, whose angles at the base are to be double the angle at the apex, or summit, is, according to Euclid, and under the supposition that a given line is in question. Having once obtained the required form, it is easy to preserve or imitate it, by means of parallels, according to Fig. 3, where it will be seen, that the middle triangle being duly formed, the larger and smaller triangles must possess similar angles, though greatly varied as to size: the fact is, that an angle of 36° at the summit, with 72° at each angle of the base, is the triangle sought; and this may be made on any line, without going through the operation of dividing that line into extreme and mean proportion, according to Euclid's directions, and conformably to Prob. XXI. of this work; neither is any circle necessary.

All polygons which have either 2, 3, or 5 for their common multiple, may be divided mathematically; thus

the quindecagon is formed by the operation of a triangle following a pentagon; and, in general, by estimating the value of the angle given from the center of the circle, for each face, or side, of the intended polygon, an accurate hand may set off the sides with tolerable precision; thus, if eighteen sides were to be found, each would contain an angle of twenty degrees; for 20 multiplied by 18, would give 360° .

Let the line AB, Fig. 2, be taken for one side of the intended isosceles; at the end intended to join the base, measure off a line at BC, standing 72° from the line AB; then from A, with an opening equal to AB, draw the segment BC, which will cut the base at a point, to which a line being drawn from A, will give the other side of the triangle sought: or, if it be required to form such a triangle on any given line as a base, or on any given line as a center, let AB, Fig. 3, be drawn perpendicular to CD; then, from the ends of the intended base, CD, set off the line CA, DA, each making an angle of 72° , and the triangle will be such as was required.

If a point is given for the apex, or summit, let a line be drawn from that point, A, Fig. 3, in such a direction as shall cut the intended base, CD, at right angles; then, from the point A, set off on each side, at an angle of 18° , a line that will intersect the base at CD, and determine its length, which will, of course, vary according to the extent of the triangle.

In this manner an infinite variety of triangles may be made, by which the circle may be divided into any number of equal portions; but where fractions are in use, the protractor, or line of chords, ought to be very accurately and clearly divided into tenths of degrees, which cannot be done on any but large instruments. We must ever carry in mind, that the whole circumference contains 360° ; consequently we may divide it into any num-

ber of equal portions, or, at least, such as are very nearly so, by dividing 360 by the number of parts into which the circle is to be portioned; say, we would divide by 17, the result would give 21° , $10'$, $35''$, and $\frac{5}{17}$, or, by the scale of tenths, about $21^{\circ} \frac{2}{10}$, which might be set off in any of the foregoing modes, from the center to the circumference of the circle; the difference would be so small, as not to affect the division materially.

I had once a semicircular protractor of brass, about a foot diameter, which was graduated to quarters of degrees, with great accuracy; but curiosity led me to try how far it might be improved by an index extending six inches beyond the protractor, moving on an half-inch pivot at the center of the instrument, and slit for about half an inch broad, where it moved over the gradations of the protractor's edge, so as to shew about five degrees thereon; the end or sweep of the index was graduated to tenths of degrees. I found the principle to be unexceptionable; but, the index being made by a black man, who had not the least idea of mathematics, and not having the means of graduating it accurately, I could not depend on its operations. The index lifted off from the center, so as to be out of the way of injury, and to pack with the protractor, with whose diameter its length corresponded.

The pivot was bored through to very near the extent of its own diameter, by which means the center of the protractor could be applied with exactness to any given point.

As an instrument of the foregoing description is invaluable, where great precision is required, and that our workmen are capable of giving it every requisite exactness, I recommend to all whose works may be curious or minute, to have such a protractor at hand.

It may be proper to observe, in this place, the joint

foot-rule, in a common case of mathematical instruments, called the *sector*, is capable of dividing the circumference of a circle into any number of parts, from four to twelve. Close to where the two sides fit, when the *sector* is doubled together, is a line of figures marked pol. meaning polygon. The quarter of any circle being taken with your compasses, let the sector be opened, so that their points may reach from Fig. 4 on one limb, to Fig. 4 on the other limb. If you wish to know what span of your compasses is requisite to measure the seventh part of the circumference of that circle, of which the fourth part is ascertained, take the distance from Fig. 7 on one limb, to Fig. 7 on the other, and that span, or opening of your compasses, will be found to divide the circumference into seven equal portions, and thus of any other number of divisions, from 4 to 12, in the same manner.

Observe, that, by again dividing the portions set off, almost any number of equal parts of the circle may be ascertained.

The term *sector* also applies to a portion taken out from a circle, by two lines from its center subtending a segment of the circumference, as A, B, C, in Fig. 4 and 5; and this, indeed, is the manner in which the instrument just described acts. Thus a quarter of a cheese may be considered as a sector.

PROBLEM XXXVIII.

To describe a regular Pentagon in a given Circle.

LET ABCDE be the given circle, make the isosceles triangle, CGD having the exterior angles at the base, CD, double that at the vertex, G, and describe a similar tri-

angle, DGC , within the circle; the base, CD , will prove to be the fifth part of the circumference, and, being set off all around, will divide it into five equal parts, or sides. A brief method, is to set off an angle of 72° from the center, G , of the circle, to its circumference; it will cut off one-fifth of the circumference, and give a measurement for the rest of the faces or sides of the pentagon.

APPLICATION.

The preparation for this problem was contained in the preceding, and its solution is here given according to Euclid; though in brief terms. I prefer the method contained under Fig. 3 of the last problem, thus: from a given point, draw a line through the center of the circle you wish should be divided into five equal parts; from that point set off lines on both sides, each at an angle of 32° ; they will cut the circle at those points that terminate the base of the isosceles triangle, and give a fifth part of the circumference, by which measure the rest of the sides of the pentagon may be determined.

Architects who have to design, and builders who have to raise edifices of various forms, will very frequently be employed to construct pentagonal temples, &c.; and the several manufacturers of jewellery, masonic devices, plaisterers, &c. &c. will find the pentagon occurring very often among the jobs in which they respectively are employed. If a sector be not at hand, recourse must be had to mathematical aid, which is useful on all occasions; for we too often find instruments extremely defective, especially after being much used; whence the points become loose, the figuring is rendered inarticulate, and the several points of measurement, or direction, are, at least, extended, if not obliterated, by the reiterated application of the compasses; and all this will

happen under the best management: besides, all instruments made of ivory, or of wood, are extremely apt to warp, particularly if left in a warm room; and every deviation, however apparently trifling, occasions a serious difference in the work.

This conviction, as to the errors created either by the original imperfection, or by the long use of instruments, shews us how necessary it is to become acquainted with every method of solving problems. Sometimes a mode of resolving a query may apply to such instruments, either wholly, or in part, which are sufficiently correct, when used for that purpose, though abounding in error elsewhere; for it is by no means singular to find a sector on which some of the scales are inimitably correct, while others are shamefully irregular; nor is this observation applicable either to inferior makers only, or to low-priced instruments. The machines used for gradating cannot but wear, or receive some bias from frequent use, and create inaccuracy, even where the utmost care and skill prevail.

But, whichever way the pentagon be made, we are to consider it as a primary figure, being no part of any other, though reducible to other forms, and easy to measure. It is the parent of all regular figures which have five for their multiple.

PROBLEM XXXIX.

To describe a regular Pentagon around a Circle.

FORM a pentagon within the given circle, as directed in the foregoing problem; and, in the middle of each side, or face, draw the perpendicular, DA, CA, BA, EA, FA, all proceeding to the center, A. The lines

drawn so as to trace the circle, DCBE and F, will describe the required pentagon about the circle.

APPLICATION.

This may be done either according to Euclid, that is, by means of Problem XXXVII. or by Fig. 3, in the application annexed thereto; but, in the latter case, the base line, AB, must be drawn at right angles to the central line, and be a tangent to the circle; or, in other words, must just touch the circle, so that the figure may be described *around*, instead of being inscribed *within* it.

By means of parallels we can extend or diminish the pentagon to any extent that may suit our purpose; having the proper angle as a guide, we never can err. I trust the student has long since learnt, that an angle measures the same, whether the lines of which it is composed, be short or long.

If it should be required to describe a regular figure of ten sides, called a *decagon*, around, or to inscribe it within a circle, the measure given by one half the base of the pentagon, would not answer for the side of the decagon; but the chord subtending a segment of 36° , would exactly measure one-tenth of the circumference.

And here it is proper to caution the student against a very common error. Many think that the division of a chord will divide the segment, or arc, into similarly proportional parts; nothing can be farther from the fact, as may be seen by Fig. 3, where it is plain, that, in the equally divided line, OP, subtending the segment, OSP, the central portions, X, Z, cut greater quantities of the segment than fall to the share of the two outer portions, IT, and in Fig. 4 it will be seen, that where the segment is equally divided, the lines drawn from the center to

those divisions, will cut the chord, OP , into unequal portions, of which the central will be the smallest; therefore, when we measure an angle, we should always draw a line across to a given point on each, equidistant from the angular point, as HI and IE , Fig. 5; for oblique lines, such as parts of circles, which must either approach to, or recede from a right line, will not give the true measurement. Thus, in Fig. 5, the angle, A, B, C , cannot be equal to the angle, CBE , though the subtending line should be of the same length; for the line AC being more remote from the angular point than the line CE , should, to subtend the same angle as ABE , reach to D ; consequently the division of a pentagon into a decagon could not follow from dividing its side into two equal portions; the *angle* must be divided, and the chord subtending the arc of the sector thus formed, will be the side of a decagon, answering to the same circle as the pentagon from which it was derived.

PROBLEM XL.

To inscribe a Circle within a regular Pentagon.

BISECT two sides, AB, BC of the pentagon $ABCDE$; the point F , where two perpendiculars drawn from the bisections at G and H , meet, will be the center, whence the circle is to be inscribed with the distance FH .

APPLICATION.

This problem is extremely simple, and its uses must be considered as combined with the application of Problems XXXVIII. and XXXIX. It, however, shews us how to extract or ascertain the fifth part of its own area,

and to find its own center, both which are occasionally useful; for instance, if after the walls of a pentagonal temple were finished, the owner should wish, in lieu of a flat roof, to arch the building over, or to cover it with a truss roof, of five faces, or sides, corresponding with the building, by this problem, the center of each face would be ascertained, as well as the center of the roof.

PROBLEM XLI.

To describe a Circle about a given Pentagon.

BISECT any two or more angles in the pentagon, A, B, C, D, E, and the union of the bisecting-lines, at F, will give the center of a circle to be drawn with the distance AF, or BE.

APPLICATION.

The application of this problem must be taken in the aggregate with all that has been said on the subject of pentagons.

PROBLEM XLII.

To describe a regular Hexagon in a Circle.

THE radius of the given circle, is a chord which will divide the circumference into six equal parts; therefore draw the diameter, AB, through the center, C, then with the radius, AC, draw from A the segment DCE, and from B draw the segment FCG, which will finish the division of the circle into six equal parts.

N. B. Draw the lines AD, DF, FB, BC, GE, EA;

APPLICATION.

The radius wherewith a circle is drawn, will lay six times within its circumference, though the diameter will lay only in one position, which is, through the center. This application of the radius is of great convenience on many occasions, and particularly in the formation of hexagons, which may, by its means, be both inscribed and described on circles: in the latter instances they are to be treated exactly the same as directed regarding the pentagon, where, by means of parallels, similar figures of any extent may be readily obtained.

PROBLEM XLIII.

To describe a regular Quindecagon in a given Circle.

DESCRIBE the pentagon, A, B, C, D, E, as before directed, then from each of those points, in succession, describe the equilateral triangle, AFG, with a line containing two portions of the hexagon for its measurement; that is to say, as from G to F in the foregoing problem. The five segments of the circle contained under the pentagon, will again be divided, by the several triangles, into three equal portions, making in all fifteen equal faces or sides.

APPLICATION.

This is a mathematical division of the circumference into fifteen equal portions, which may be useful in ornamental work; the problem is, however, given more with the view to shew what may be done by the combination, or revolution, of one figure upon another, than from any

opinion of extensive utility; for the same effect would have been produced by trisecting (that is, dividing into three equal parts) each of the five segments.

PROBLEM XLIV.

To change a given Hexagon into an equal Parallelogram.

LET A, B, C, D, E, F , be the hexagon, produce the lines AF and BC , so that the line DE may meet them at G and H ; draw the perpendicular GK and HL , so as to meet a line drawn through the points F and C , in K and L ; the parallelogram, K, G, L, H , will be then found, and its various triangles seen, the two lateral right angles at the sides being, when joined, equal to either of the other five.

APPLICATION.

The hexagon is by no means an uncommon figure, and may often come under the surveyor's and builder's observation.

We have no shorter mode of commuting the hexagon into a parallelogram, or of estimating its contents, than by extending one of the sides, as ED , in the figure attached to the problem, to GH , so as to be three times its former length; taking that for the base, and half the diameter for the depth, forms the desired parallelogram. Those who make, or cover umbrellas, though they may not know this rule in the form of a problem, nevertheless follow it in their practice; in that business, as well as in many others in which triangular parts are to be fitted, the cloth, &c. must, if speed and economy are of moment, be cut out in a series of isosceles triangles, whose

points and bases are alternate, as shewn by the lines in the parallelogram H, L, G, K; their position shews how readily they may be cut out, and their numbers answer to the sides of a hexagon, the two end half-sides being joined to complete the sixth side.

PROBLEM XLV.

To change a given Parallelogram, whose Base is equal to Three Times its Height, into a regular Hexagon.

DIVIDE the base line, A B, into three equal parts, and at each end of the line C D, at E, and at F, mark off half of one of those portions, that is to say, one-sixth of the said line C D; draw the lines A G and B H, then draw K G and L H, which will cut the former lines at G and H, so as to give the upper part of the hexagon G H; draw L E and K F, which will finish the lower parts, and exhibit the hexagon, G H F K L E.

APPLICATION.

Although the above proportions are requisite to describe a regular hexagon, or a flat surface, yet, in objects that have height as well as circumference, such as an umbrella or a pyramid, the angles must be more acute, and the spread must be regulated by the number of angles or gores that may be necessary to give the proper cant or slope for throwing off water; but, to make the figure regular, all the angles should be similar. To prove this, let a piece of paper, in form of a regular hexagon, be divided into six equal triangles, by three diameters from the six points, all intersecting at the center; cut

them asunder, so that each triangle be separate; no art can make them form a pyramid, without either bending or overlapping; but, if the bases be narrowed, by taking off a slip on each side, all the way up to the apex, or summit of the angle, as AB , AC , in Fig. 2; they will, when joined at the bottom, rise in their common center, and form a pyramid, whose altitude will be augmented, if more be taken from the base.

This proves not only what was before asserted, but that wherever triangles are used to form any regular figure having central elevation, they must not be equiangular; that is, they may be acute at their summits, but the angles at their bases must be more than 60° each.

Where there are many gores, as in the boot, or round ends of a marquee, they are very narrow and numerous; they are made by cutting the canvas diagonally, as from D to E , in Fig. 3, and joining them, as in Fig. 4; the small angular surplus at the end of each gore being cut off, to render the whole of the lower rim, or edge, equidistant from the center, and give them a circular inclination, or spread.

PROBLEM XLVI.

To change a regular Pentagon into a Parallelogram.

IN the pentagon $ABCDE$, draw from the center, F , a perpendicular, to fall on the center of its base, DC , at G ; continue GC to H , so far as to be five times the length of GC , and with that line as base, and FG for the side, describe the parallelogram $FGKH$, which will be equal to the pentagon $ABCDE$, for there will be five parallelograms, each equal to one-fifth of the pentagon.

APPLICATION.

This commutation not only affords the means of ascertaining the contents of the pentagon, without any triangular division, but it enables us to form at pleasure a pentagon from a parallelogram, on the same principle as relates to the hexagon, of which notice has been taken in the preceding problem. But, attention must be paid to the point before stated, *i. e.* that if the center of the pentagon is to be raised, as in the tent, or umbrella, the bases, *i. e.* the length of the sides, must be reduced, so as to leave a space between the corners; otherwise the figure, or object, will remain flat; and, as in a figure of six sides, each side stands at an angle of 60° with its neighbour; and that their aggregate being 60° multiplied by 6, gives 360, the number of degrees in a circle; so a figure of five sides will give an angle of 72° , between the contiguous triangles, making in all 360° ; 72° being multiplied by 5. This proves that the angles at the base of each side should be more than 72° , else there could not be any central elevation: the more we cut from each side of the base, observing to draw the lines carefully to the apex, or summit, which is never to be altered, the higher will be the peak: thus, in all ornamental pyramids, each side has a narrow base, whence the figure appears more raised and taper. See Fig. 2.

PROBLEM XLVII.

WE now come to the most beautiful problem to be found in the earlier branches of mathematics, *viz.* the square of the hypotheneuse, or longest side of a right-

angled triangle, is equal, in contents, to the squares of the two other sides added together. This forms the subject of the 47th of the 1st book of Euclid; it is a problem of much importance, and cannot be better explained than by the following extract from that author:

“ On each side of the right-angled triangle, BAC , form squares, as $BAFG$, $AHCK$, and $BCDB$; then, from the point A , draw AL perpendicular to DE , and join FC and AD ; draw also AE and BK .

“ Observe, that the inside of the original triangle is common to all the squares, and measures with each of them successively; this, it is obvious, causes no irregularity or inequality; for, whatever part is taken in the illustration by one, is also taken by, and ultimately relinquished by, each of the other squares.

“ Now, the angle DBA is equal to the angle FBC ; the sides AB , BD are equal to the sides FB , BC , each to each; the base AD will be equal to the base FC , and the triangle ADB will be equal to the triangle FBC ; but the parallelogram BL is double the triangle ABD , for they are on the same base, BD , and between the same parallels, BD and AL . The square, $BAFG$, is likewise equal to double the triangle, BFC , they being both on the same base, FB , and between the same parallels, FB and GC ; therefore the square, $BAFG$, must be equal to the parallelogram BL . After the same manner, the square, $AHCK$, is equal to the parallelogram CL , and therefore the two parallelograms, BL and CL being united, are equal to the squares of the other two sides of the right-lined triangle, ABC .”

N. B. The solution depends on principles which form a main support to mathematics in general.

APPLICATION.

This famous problem seems to concenter in itself the greater part of what is contained in the preceding; for there are few matters hitherto treated of, which do not come into either its formation, or its solution; hence it has been designated "the pons asinorum," or the ass's bridge; implying that all who could correctly state the problem, and account for its demonstration, must have made such a progress in mathematics, as entitled them to be exempt from the imputation of ignorance. A careful inspection will satisfy that the inference is by no means erroneous.

We are indebted to this problem for the means of proving the correctness of perpendiculars, as well as for the bases of all the calculations dependent on the angles contained in various segments of circles; for it was, doubtless, owing to the perception, that the angle contained in a semicircle, must be a right angle, that research was originally made regarding the measure of angles contained in the other segments of circles; whence has arisen a branch of mathematics to which many of our most important arts, navigation, and optics, for instance, are chiefly indebted. Nor should we ever have been able to ascertain the center of a circle, of which only a small segment were given, but from the knowledge of the foregoing facts. In truth, we seem to be supported in all the superior mathematics by this great pillar, of which all the component parts are correctly demonstrable, as must already have been satisfactorily proved by the preceding problems.

Every profession, wherein measurement or squaring of angles at any time occur, must derive considerable aid from the explanations given, for they are applicable not

only to the scale and compass, but to arithmetical computation. Thus, if a house-carpenter would know what length of rafter A B, Fig. 2, would be required for a roof, to be supported at one end by a wall, A C, of 30 feet high, the room, C D, being 40 feet broad, he would first find the square of 30, viz. 900, and the square of 40, viz. 1600, which two squares being added together, would make 2,500 square feet. Now, according to this problem, "the square of the hypotenuse is equal to the square of the other two sides of a right angled triangle;" therefore, as the wall, A C, is perpendicular to C B, the angle A C B must be a right angle, to which A B, that is to say the rafter, is the hypotenuse. Let the square root of 2,500 be found, which is 50 (for 50 multiplied by 50, will make 2,500) and that will be the length of the rafter.

And thus any one of the sides of a right angled triangle may be found, if the two other sides are given, by ascertaining the difference between the squares of the two given sides, and extracting the square of the side to be found, which will be its proper length. Thus, if the base, measuring 40 feet, and the hypotenuse of 50 feet, are found, deduct the square of 40, *i. e.* 1,600 from the square of 50, *i. e.* 2,500, the residue will be 900, of which the square, viz. 30, will be the third side.

In that part of this work which treats of surveying, we shall find this problem blended with the several operations in various shapes.

PROBLEM XLVIII.

To ascertain the Contents of a Circle.

It is well known, that the contents of the circle are not to be found by any mechanical operation; figures have,

indeed, brought the calculation of its superficial measurement so near, that our most intricate computations, particularly in astronomy, scarcely afford any proof of inaccuracy. Let it be understood, however, that the squaring of a circle yet remains among the desiderata of mathematics. Few artizans require absolute perfection in this particular, and persons of all professions content themselves with the knowledge, that the diameter of a circle is as 7 to a circumference of $22\frac{6}{7}$; that is to say, if a line drawn through the center of a circle measures 7 inches, the circumference will measure $22\frac{6}{7}$ inches. It has been established, by the best authorities, that, for every inch in the diameter of a circle, $\frac{886}{1000}$ of an inch should be applied, as a side on which a square should be formed; thus, if the diameter of a circle be 10 inches, multiply the number 886 by 10, which will give a total of 8,860, of which the left hand figure (8) implies integers, or whole numbers; and the figures on the right of the dot, imply decimals, or thousandth parts; therefore, the length of the line should be, in rough numbers, 8 inches, and $\frac{7}{8}$ of an inch.

A square, whose sides will each measure $8\frac{7}{8}$ of an inch, will nearly equal the contents of a circle of 10 inches diameter, or describe a right angled triangle, (Fig. 2) of which two sides shall each be equal to a radius, or half diameter of the given circle; then the long side, called the hypotenuse, will be equal to the side of a square, which will contain an equal surface with the given circle.

So, if AB be the radius of the circle ABC, let the two sides, DE, EF, of the right angled triangle, DEF, be equal to AB or AC, and stand at right angles; then the line DF, called the hypotenuse, will be the side of a square, equal in contents to the circle ABC.

APPLICATION.

Much has been said on this subject, to enable the student both to ascertain, with tolerable correctness, what the mean measurement of a circular area may amount to, and to apply that measurement towards commutations of various descriptions; but I must particularly remark, that the operation should be conducted with peculiar delicacy, else the product will by no means establish the credit of this problem. The diameter must be very accurately taken, and the whole process ought, indeed, to be well examined, and proved to be correct, before any reliance is placed on its issue. We are easily deceived as to the exact points where a diameter cuts the circumference; and, as a very small difference in length makes great variation in the work, it has always been my rule to guard against the errors occasioned by lines, by laying aside the pencil, and measuring, by means of a pair of hair compasses applied to the thin edge of a protractor, very carefully placed along the diameter of the circle to be measured. By this method, something like precision may be attained, especially if the points of the compasses are not pressed into the paper, but suffered to make only such slight marks as their own weight occasion; such will not be very distinct, but, to a keen eye, will be sufficient, while the faults generated from pressing the points into the paper, will be avoided: a little experience will satisfy the young mathematician, that even a slight perforation of one sheet, of moderate thickness, will, in work requiring great exactness, create much embarrassment. I have already said much respecting fine points and slender lines; but, I must repeat, that, in this very delicate operation, too much attention cannot be paid thereto.

This problem enables us to estimate the contents of pillars, or other such solids; to calculate the quantity of water raised by pumps, or passing through pipes or other channels; to ascertain the superficial measurement of circular plots, &c.

Thus, if a pillar of brass were to be estimated, which should be 20 feet high, and 24 inches diameter at the base, 26 inches at its swell, or thickest part, which should be eight feet from the base, and that the diameter should be 18 inches at the top, the mean diameter between 24 and 26 would be 25, which being taken for the diameter, as far as 8 feet high, would, when squared, furnish the solid contents of that part of the pillar; then, as the medium between 26 and 18, would be 22 inches, that would be the diameter to be squared for the residue, 12 feet; and thus the whole contents of the pillar could be easily known. Its weight would appear by taking the weight of ten solid inches (or any other quantity convenient for calculation) say at 5 lb.; the number of solid inches being known, and computed by the above standard, which gives 8 oz. to the solid inch, the aggregate weight of brass in the pillar would be established.

In regard to pump-makers, much might be said; but it would not answer the limits of the present work, to enter into the detail of that branch of mechanism. On that subject, I must refer the student to another treatise I am about to publish, wherein every matter relating thereto will be amply detailed.

PROBLEM XLIX.

To form a given Circle into a Triangle.

DRAW the diameter, AC, through B, also the baseline, CD, touching the circle at C, but in length equal to

three diameters, and one-seventh; then, from the center, draw B E, and a triangle will be found equal in contents to the given circle.

Say that a circle be seven inches in diameter, then the base-line will, as above directed, be 22 inches.

N. B. The circle may thus be changed, by means of the triangle, into any form of equal contents.

APPLICATION.

Although this problem may occasionally be of service in commuting circles to triangles, or, by a second operation, (namely, that of doubling up the triangle, vide Problem XXII.) into a parallelogram, yet its principal utility seems to depend on inversion, that is, in changing a given triangle into a circle. Many instances may occur in which this may be useful, such as changing the form of a weight or measure, ascertaining the quantity of the materials requisite to build, floor, or roof, in a circular edifice, and other similar matters, of which the principles have already been described in various parts of this work.

I shall give an instance, in which the manner of working this problem will be exhibited.

It is required to ascertain the exact quantity of masonry in a house, of which all the doors and windows are circular at their tops, as ABC, in Fig. 2. The shortest way is, certainly, to consider the walls as being entire in the first instance, and to multiply the breadth of each face, both by its height and by its thickness, as if the wall had no breaks or openings; then, ascertaining the cavity made by the several doors and windows, to deduct their amount from the mass. The square parts are easily computed, but the semicircles must either be squared or triangled; the latter is easy, and unobjectionable. As the whole circle would give the radius for the alti-

tude, and $3\frac{1}{7}$ diameter for its length, take the same altitude, and half the above base, that is 1 diameter and $\frac{8}{14}$, and that triangle will be equal to the semicircle; or take the whole of the base, as before, and only half a radius for the altitude.

Suppose it were required to ascertain what number of cubic feet would be excavated by a semicircular tunnel of 14 feet diameter, and 170 yards in length; here the shortest way is to consider the semicircle as a whole circle, and to take only half the distance, viz. 85 yards for the length; then, 7 feet being the radius, and consequently the altitude, and three diameters and a seventh being the length of the triangle, which would be 44 feet at its base, 44 multiplied by 7, would give 308 for the superficies; and that multiplied by 255 feet, which are equal to 85 yards, would give 78,540 solid feet of excavation; which, divided by 27, the number of solid feet in a solid yard, there would be 2,808 solid yards, and 24 solid feet.

In this manner the problem may be applied to many branches of business.

PROBLEM L.

How to draw a Spiral Line round a given Point.

MAKE the line AB at pleasure, bisect in C, round which, as a center, the spiral line is to be drawn; with a small opening of the compasses, describe the semicircle DE, above the line AB; then move your compasses to D, and draw, below the line, the semicircle EF; again change your compasses to C, and with the distance CF, draw above the line the semicircle FG; this may be repeated as often as may be found proper, observing that

the compasses will be extended one distance more for every semicircle drawn, and that the center C is for all drawn above, while the center D is for all drawn below the line; which might be previously marked off into as many divisions as there are to be turns of the spiral; though such preddivision is not absolutely necessary.

APPLICATION.

We often see mounds, whether natural or artificial, around which walks are carried, from their bases to their summits, in a spiral manner, which is, indeed, the only means of ascending great steepes, whose forms are conical. Many of these walks are made without their line being first traced out, according to a regular plan, but they never please the eye, nor is the ascent, in such case, regular. The declivity of a conical mound may be as regularly divided, and set off into either right or curved lines, as could be practised on a level plain, as is fully proved by Problem V. which shews how to divide a right line in the same proportions as another line, though not of equal length with that by which it is divided. By inverting this operation, the longer line, such as the side of the hill, will be divided exactly the same as the level line.

To draw a spiral round the hill, ABC, Fig. 2, let the side, CB, be divided into as many equal portions as there are to be spirals around it, as D, E, F; put down stakes at those spots, and divide the opposite side into two equal portions, as between HI and IL, first setting off half such a distance, viz. HA, from the bottom.

Now draw the line CH in front of the hill, and let the dotted line, HF, pass at the back; again draw FI in front, then the dotted line IE at the back; again from E

to L, and dot back from L to D; the line DB in front will nearly reach the summit. By this means the ascent will become steeper as you get nearer to the top, and the spiral will be far less regular than if the whole were parallels. The latter method is assuredly preferable, and can be carried into effect by measuring similar angles at the commencement of each semi-spiral, or, by preserving the parallel, by means of a rod reaching from the lower spiral to that which is making. See Fig. 3, in which the rod is shewn by dots, as a perpendicular at each end of every line of ascent.

Spirals are to be found in the ornaments of columns of various orders, under the designation of volute; they are formed on the principle of Fig. 1 in this problem.

Spirals around pillars are described according to the plan laid down for the ascent up a mound, (see Fig. 3); that is, at equal distances; they answer well in this situation; for, as the pillar is nearly cylindrical, the several revolutions will be so nearly parallel, that the difference would not be perceptible to the observer.

PROBLEM LI.

To find the Center, and the Two Diameters of an Oval or Ellipsis.

LET ABCD be an oval, in which draw at pleasure the parallels BA and CD, bisect them in F and G, then draw the line FG, the middle of which, X, will be the center of the oval.

Describe the circle HKLM, and, where it cuts the oval at H and K, draw the line HK; from the middle of which, produce the perpendicular NO, passing through

the center, and being the greatest diameter; the lesser diameter will be found by the perpendicular VY , drawn through the center, X .

N. B. The short diameter of an oval is sometimes called its conjugate, and the long diameter its transverse diameter.

APPLICATION.

This problem not only relates to ornamental work, but to various parts of edifices, windows, sky-lights, &c. being frequently oval, as are also many flights of stairs; that is, they are carried round an oval *newel*, or well. In such case, the corners of the apartment are generally filled up to correspond with the shape of the newel; the ceiling also is laid out in a suitable manner.

If the carpenter and plaisterer are ignorant of the object of this problem, they cannot fail to disfigure every part of such an undertaking; for neither can the steps be drawn truly to the center, nor can the compartments of the ceiling be justly divided,

PROBLEM LII.

To draw an Oval upon Two equal Circles.

ON a given line, AB , draw two circles, intersecting each other, by the circumference of one passing through the center of the other; at the two points of intersection, C, D , draw the perpendicular QG , and draw the tangent EF ; then from the center, G , of the tangent, EP , draw the line GH , passing through the center O of the circle; draw also KL parallel to CD , passing through the center O ; now, with an opening of your compasses, equal to KB , placing one foot at H , and the other at G (where it will touch on the line CD , below the center of the two

circles); draw the arc, HN , which will combine with the extremities of the two circles, and afford a good oval. The operation is to be repeated, for the completion of the opposite side, only so far as relates to ascertaining CF , a line similar to GH , also parallel thereto, which gives the center Q of the segment to be formed from the other end of the line CD .

APPLICATION.

Ovals being of various proportions, the mode of making them must often depend on the taste of the artist, or on the space to be occupied; the latter consideration must decide as to the greater or less breadth, in proportion to length. The above directions will, however, serve for general purposes.

An oval drawn upon two equal circles, is a very common kind of ellipsis; but, in general, it has a heavy appearance, the depth being too great for the length.

Much cannot be said as to the utility of this, or any other kind of oval, except that it is a very common ornament, and that, consequently, the formation of this figure, in all its varieties, should be well understood by all persons whose avocations subject them to be called upon for either the designing or the executing of such forms. To the maker of frames for looking-glasses, &c, this problem is peculiarly interesting.

PROBLEM LIII.

To draw an Ellipsis or Oval on Two separate and equal Circles.

LET two circles be drawn, touching each other, and through the point of contact, A , draw the perpendicular

BG; draw also the tangent DE, from the center of which make the line, FG, passing through the center of the circle at H; then taking a diameter and a half, viz. from H to M, for a radius, and placing one foot of your compasses at F, describe the segment FK from the point C; the other side will be completed by inverting the operation, as is described in the foregoing problem.

APPLICATION.

This ellipsis has much the advantage, in regard to appearance, over that projected in the last problem. If the oval should, on account of any peculiarity of situation, be still more elongated, the circles may be removed to a suitable distance from each other; and the centers from which the segments joining them are drawn, should be in the same proportion removed from the center A, along the line BC. See Fig. 2.

The ellipsis will appear yet lighter, and more fanciful, if the circles are somewhat reduced in size, whereby the ends of the figure will seem to draw more to a point, and partake rather of the appearance of a Gothic oval, constructed by the intersection of two segments.

PROBLEM LIV.

To describe an Ellipsis of a true Egg Form on Two separate and unequal Circles.

DRAW on the line OP a circle, and with an opening, equal to half its radius, draw another smaller circle, touching the first, and on the same line; then, from the point A, i. e. at the third of the diameter of the larger circle, draw the perpendicular AB downwards, equal in

length to two diameters of the larger circle; from B, as a center, draw the segment CD, which will give a true egg-like form to that side: to finish the oval, the operation must be inverted, as before, by the continuation of the line AB through the greater circle.

APPLICATION.

Although this form may be applicable to particular occasions, we have few regular, or general uses, to which it is applied, if we except the egg and anchor mouldings, in which it occasionally appears very conspicuous. In some turnery it forms a principal feature, such as the tops of nine-pins, bell-handles, and a few articles of that kind, which, however, are mostly formed from fancy, rather than from any particular measurement, or model. The human countenance is nearly of this shape, but may be more exactly imitated by still further diminishing the lesser circle.

PROBLEM LV.

To describe an Ellipsis, or Oval, on Three equal Circles.

ON the line AB, draw three equal circles, so that the two end ones shall be half a diameter asunder; then draw tangents above and below, as at CD and EF, join C to E, and D to G, drawing the lines through the points where the center circle cuts the others; draw also the perpendicular GH, through the center of the middle circle; then draw the oblique lines DK and EL, passing through the centers of their respective circles; with the opening HL, equal to a diameter and three quarters, from the center, H, draw the segment LM. To complete the figure, reverse the latter part of the operation, as before shewn.

APPLICATION.

This makes by far the handsomest oval, and in long narrow bands, or in slight cornices, give a much lighter air than any of the ovals formed on two circles; but it is not so appropriate as them for chain-work.

PROBLEM LVI.

To describe the Figure of a Heart.

DRAW two circles, touching each other, through the center A; draw the tangent FC, and through the center B, draw the tangent ED; from F, with the distance FG, draw the segment GK, and from E; draw the segment HK, which will complete the desired figure.

APPLICATION.

This figure is purely ornamental, and is generally formed according to the workman's fancy. I have inserted it as a problem, because the proportions given are beautiful, and that it enables the designer, or labourer, to draw the figure on any scale.

I should here remark to the student, that an acquaintance with mathematics will enable him to form a thousand ornaments, completely symmetrical, and to transfer them from the paper, on any scale, with justness and facility; in fact, no limits can be placed to the exercise of his fancy.

PROBLEM LVII.

To describe a regular Serpentine Figure on a given Line.

ON the line AB , describe the semicircle BC *above*, and, with the same opening, describe the semicircle CD *below* the line; continue this at pleasure for a single line; but, if a double line is to be described, after drawing, as in Fig. 2, the semicircle BC above the line, draw its parallel, AE , at any distance required; then, from E , carry on another semicircle, with the small opening of the compasses, below the line to F ; then, from C , with the large opening, describe CG , and so on, using the larger and smaller openings alternately; observing, that each comes alternately above and below the line, the smaller circle being always next to it, and both being drawn from the same center, else they could not be parallels.

APPLICATION.

The painter, the cabinet-maker, the carver, the brass and iron-founder, the plaisterer, the gardener, and various other professions, all use this problem, to construct single or double serpentes. Where the work is extensive, a model made of thin board, or of tin, called a stensil, in which, not only the outline, but any subordinate pattern, may be pierced, is assuredly the quickest and most exact mode; and this equally in square work, as in a succession of segments seen in Fig. 3.

The serpentine-pattern is often found running in angular directions, or turning in among various figures; when such happens, in lieu of any increase of difficulty,

we generally find some assistance from the many guiding points with which such patterns present us, as in Fig. 4.

PROBLEM LVIII.

To find the Center of the Two Segments of a Gothic Arch, as also their common Center.

FIND the two centers, D and E, from which the segments AB and BC were drawn; join those centers by the line DE, and that being bisected in F, will give the point to which a perpendicular, drawn from B, will divide the arch into two equal and similar parts.

APPLICATION.

Masons, carpenters, glaziers, &c. are sometimes perplexed, when they have to repair Gothic edifices, how to fill up vacancies occasioned by violence or decay, as well as how to fit in doors or windows, and to divide any ornament thereon, into just proportions, for want of a rule whereby to ascertain the origin of the curves, and to find the center of the arch. By this problem that difficulty is done away, and the workman is enabled to place every thing in its just situation. See also Prob. XVII.

PROBLEM LIX.

To measure a Solid whose Sides are all unequal.

FIND a mean proportion between the thickness O, O, and the breadth, S, S, of the solid, ABCDEFGX; this will give a square, whose surface will be equal to the

area contained in the base, $ABCD$; do the same with the other end, $EFGX$; the lines EB , FA , GD , and XC , being also of different lengths, let the whole be added together; then dividing their aggregate by four, (or by the number there may be of angles, if the solid be a pentagon, &c.) and the product will be the medium of their respective lengths. This, however, supposes the surfaces to be even, for in very irregular solids an accurate measurement cannot possibly be taken. If the base E , X , F , G , vary in size from the other base, $ABCD$, their medium must also be taken.

APPLICATION.

This rule is of such general use, especially among stone-masons, and those who deal in marble or in timber, as scarcely to require any observation regarding its importance. Hay-stacks, marl-pits, &c. may all be measured by this means. Where large irregular cavities occur in the latter, they must be separately calculated.

AXIOMS.

THE following Axioms, though partly contained in the foregoing pages, are here introduced to the student's notice, under the hope they will aid in removing any misconception, and afford a clearer insight into the subject at large. See the several figures.

1. If circles touch each other, in any part, they cannot have the same center.

2. If from any point in a circle more than two equal lines can be drawn to the circumference, that point must be the center of the circle.

3. A circle cannot cut another in more than two points, nor can it touch another in more points than one, either internally or externally.

4. The diameter is the greatest line that can be contained in a circle; and those lines which are nearest to the center, must be greater than such as are nearest to the circumference.

5. If any right line touches a circle, and a line be drawn from the center of the circle, to the place where such a tangent touches the circle, such line drawn from the center, will be perpendicular to such tangent.

6. If any line touches a circle, and a perpendicular be drawn from the point where it touches, through the circle, the center will be somewhere in that line.

7. Angles that are in the same segment of a circle, must be equal to each other.

8. If a segment, less than half of a circle, be measured

off by two lines from the center, the angle they contain must be double the angle contained in two lines drawn to the same points, from the center of the circumference opposite to such segment.

9. Figures are said to be *described* around others, or *inscribed* within them, only when their points are touched thereby.

10. Figures are equal, only when they measure an equal surface, without regard to their forms; and they are similar, only when their forms are alike, though perfectly differing in extent or measurement.

11. All similar right-lined figures are to each other according to their bases, and all similar ovals, and circles, are to each other according to the square of their diameters.

12. The altitude of any figure is measured by a perpendicular line drawn from the top, or vertex, to the base; or, (if the vertex be not over the base), to a continuation of the line of the base, until it may meet the perpendicular drawn from the vertex.

13. Solid parallelipedons, prisms, pyramids, cones, and cylinders, standing on equal bases, and between equal parallels, must be equal to each other, in their several classes, respectively.

14. Circles being to each other, according to the squares of their diameter, so similar polygons, inscribed in circles, are to one another as the squares of the diameters of the circles.

15. Every prism, having a triangular base, may be divided into three pyramids, equal to one another, and having triangular bases.

16. Every cone is a third part of a cylinder, having the same base, and equal altitude.

17. Cones and cylinders, of the same altitude, are to each other as their bases; but similar cones, and cylin-

ders, are to one another in a triplicate proportion of the diameters of their bases.

18. Cones and cylinders, being upon equal bases, are to one another as their altitudes.

19. Lines that are not parallel, must, if they be produced, or carried on, meet at some point to which they both have a direction, and at that point they will form one angle; but if they cross each other, they will form four angles.

20. If a line cuts another at right angles, the four angles will all be right angles, each measuring ninety degrees, or a quarter of a circle; but, if one line cuts another obliquely, then there will be two acute, and two obtuse angles, opposite to each other respectively.

21. Any number of lines meeting, or intersecting, at the same point, and forming any number of angles, so as to exceed half a circle, must have a total amounting to 360° ; but any number of lines meeting on any part of a line that subtends a semicircle, will form a total of only 180° .

22. Any two lines falling at the two ends of a chord, subtending an arc equal to half a circle, must form an angle of 90° ; and the two ends of the angle made from the extremities of such chord, will be less in proportion, as the segment of the circle subtended, is less than half a circle.

23. When an angle is formed, by two right lines, they will both preserve that angle from their outset, at the point of union, to whatever length they may be carried. Thus, if an angle be such as to cause two lines to be one inch asunder at a given distance, measured on each, from the point of union, when they may have proceeded double that distance, they will then be two inches apart.

24. Two perpendiculars cannot fall on the same point in any given line, so as to be perpendicular to that line;

but, two or more lines may intersect at the same point, and be perpendicular to each other respectively; for all such as form angles of 90° , must be perpendicular to the lines they so fall upon, or intersect, as a to b, and c to d.

25. All lines that are parallel to any one line, must be also parallel to each other; thus, the two lines running the length of the bottom of a square chest, or other prism, will be parallel to those running lengthways at the top; each is parallel to that which is next to it, of course all must be mutually parallel.

26. So, all lines that are perpendicular to any other line, must be parallel between themselves.

OF

THE INSTRUMENTS

REQUISITE FOR

PRACTISING MATHEMATICAL OPERATIONS,

TO THE

EXTENT LAID DOWN IN THIS WORK.



A PROTRACTOR, which may be of brass, in which case it is generally semicircular, and has a line of chords on the straight bar. Those made of ivory, or wood, are much better, as, besides having two lines of chords, they generally are furnished with various sized scales, extending from 20 to 60 measures in the length of a foot; whereby work may be set off to any proportion. By applying the mark on the plain side to any point on a line, any number of degrees may be set off, referring to the edge of the protractor, on which they are marked: thus angles of any extent may be made or measured.

A Sector, either of wood or of ivory; it is usually a foot ruler, doubling in the middle, and having various scales, of which the uses do not occur in this treatise, except in one or two instances, where the requisite instructions are furnished. The inches being divided into tenths, afford a good scale for more extended works; and on the same edges there will be found a division of the foot into

nine equal parts, which are again subdivided, and supply an useful scale.

A Parallel Ruler, of wood or ivory: those with a small center-piece, forming a third slip, are extremely useful, but rather more apt to warp, and get out of repair, than the common kind.

A Pair of Plain Compasses, of which the points ought to meet very close, and be fine: these are for taking distances, measuring lines, &c.; they should work with a screw and spring, so as to take distances with extreme exactness; such are called *hair-compasses*.

A Pair of Drawing-Compasses, of which one of the steel legs draws out, and may be replaced either by a crayon-leg, made to hold a pencil, or a pen-leg, which is of steel, with springs and screw, for drawing circles with ink, or a wheel-leg for making dots.

A Line-Pen, made also with steel springs and screw, for drawing lines with ink: this in general unscrews in the middle, and has a fine steel point fixed to one part, for pricking off measurements, &c. by which means work will be done more accurately.

A pair of Bowes's Compasses, which are small, and only intended for drawing small circles in ink, are generally given with the case.

A Pencil is usually fitted to a small metal top in one corner of the case, and a kind of compass-screw is also given, having one end like a turn-screw, for turning screws, and the other with two small projecting studs, applicable to the small holes in the heads of the compasses, so as to unscrew and separate the legs for cleaning, or to screw them up to any degree of tightness.

All the foregoing problems may be worked with these instruments; and I strongly advise those who seek no further information in mathematics, not to burthen themselves with any others.

As, possibly, persons who may have occasion to study this Introduction to Mathematics, may likewise be intent on obtaining a knowledge of Surveying, so as to estimate the extents of farms, to inclose commons with regularity, to lay down the plan of a house and out-offices, or eventually as a profession, I offer a few tables relating to measurement; such, indeed, as ought to be familiar to every mind, but which are indispensable to all who are concerned with the soil, or with buildings.

Table of *Statute Land Measure*, (as to length).—3 feet make 1 yard; and $5\frac{1}{2}$ yards, or 16 feet, 6 inches, make 1 rod, pole, or perch; 40 pole, or 220 yards, make 1 furlong; and 8 furlongs, 1 mile.

Table of *Statute Land Measure*, (as to the square).—144 square inches, make 1 square foot; 9 square feet, make 1 square yard; $30\frac{1}{4}$ square yards, make 1 square pole, perch, or rod, (sometimes called a lug); 40 square poles, perch, or rod, make 1 square rood, containing 1,210 square yards; 4 square roods, make 1 acre; and 640 acres are contained in a square mile; an acre contains 4,840 square yards, and amounts, with the exception of a small fraction, to a square measuring 70 yards in length, and the same in breadth: this, in walking over lands, may prove a very useful guide.

I shall hereafter give a description of Gunter's Chain, though it is an instrument I never use; it ought, indeed, to be exploded, as extremely unfit for the generality of persons not versed in fractions; in lieu of it, I recommend to persons requiring only temporary substitutes, a well-stretched log-line, marked at every ten yards, by means of a coloured yarn, on which a knot should be tied for every ten yards on the line; thus, at the first ten yards, one knot; at twenty yards, two knots; at thirty yards, three knots; and so throughout; one hundred yards will answer. Care should be taken not to strain

the line beyond common tightness; that is to say, it should neither appear tense, nor loose; if managed with care, and now and then corrected, it will be tolerably perfect. But nothing can equal a chain of one hundred links, provided each link be one foot in length; this chain should be divided as directed for the line, or like Gunter's, of which a description is given in the latter part of this work. A rod of fir, ten feet in length, having the feet, with their halves and quarters marked thereon, is also necessary; and about a dozen slender sticks, like arrows; also three wands of about four feet long each, should be provided; they all should have small coloured flags of a few inches square at their tops, to distinguish them at a moderate distance, and should be shod with spikes.

The following instruments, and method of surveying, I consider as being peculiar to myself; I do not notice them here merely because they are of my own invention, but from the intire conviction that they are best calculated for such persons as are not provided with Theodolites, without which we cannot survey with perfect accuracy.

The generality of purposes, among those who are not surveyors by profession, may be answered by means such as are now pointed out; they have, at least, the recommendation of being simple, cheap, and adapted to almost every situation!

I am the more induced to dwell on these very plain instruments, and easy modes, from the experience I have had of their answering, in careful hands, far better than more intricate constructions and computations. My instrument, which I call the *Standard Triangle*, has proved itself sufficiently correct. I am sorry to say that, more than once, several surveyors, eminent in their profession, having gone over the same estate, with superb ap-

paratus, a difference of near an eighth part of the whole has appeared in their results; especially, in one instance, where the concern was large, and the measurements were taken by two surveyors; one on the part of the buyer, the other on the part of the seller!

“ Who shall decide when doctors disagree ? ”

Why, on a hint given, the two surveyors went *together*, taking, alternately, sights from the same instruments; by this means a *third* result, differing widely from the two former, was obtained, but, by previous agreement, it stood decisive! The gentlemen of the quadrant, (whose names I shall not reveal, though, I believe, the story is well known in the north), did not, like the faculty, quarrel while the patient died, but, being handsomely paid for the coalition, very prudently resolved to consult each other's field-books, rather than disagree in future!

DESCRIPTION OF THE STANDARD TRIANGLE.

Plate 9.

THE standard, A B, Fig. 1, is five feet, six inches, in length, having a strong iron spike at the bottom, to stick firmly in the ground. The instrument should be of deal, one inch thick; and two inches broad; the piece, C A, is fitted on perfectly rectangular to A B; the stay, E F, should be so made, as to render the *inside* of the triangle, E F G, of the following dimensions; C G, thirty-six inches; G F, forty-eight inches; and E F, sixty inches; or, in other words, three, four, and five feet, which will form a regular right-angled triangle, of which E F is the hypotenuse. But the instrument may be made to any size.

About D, a small hollow, three-fourths of an inch deep, should be scooped out, to allow a plumb, sus-

pended from the point G, to lay in, when the standard is perpendicular. It is of great importance that the stud, X, Fig. 2, from which the plumb is suspended, should have a very narrow neck, it being the center from which the gradations of 90° , marked on the hypotenuse, EF, are taken; it should further have a small loop of iron to receive a second line of whip-cord, about seven or eight feet long, of which the purposes will be explained. See Fig. 4.

On the reverse side of the standard, a small plate of iron (Fig. 3) should be fastened, having a small round perforation, exactly corresponding with the center of the stud and loop. The use of this is, to pass the whip-cord through, when the standard is reversed, so as to become an exact center, from which to measure off any required angles; as will be hereafter shewn, when treating of the construction and uses of the quadrant.

To survey by this instrument, requires neither great talent, nor great experience; if there be any defect in its construction, it is easily rectified, as will appear from its adaptation to the following examples, which are not numerous, but progressive, and include all that the common surveyor, or the gentleman, wishing to inspect his own estate, or to direct improvements therein, can possibly require.

I shall commence with instructions for dividing a square field into two equal parallelograms, by a line drawn through the middle.

EXAMPLE I.

To divide a square Field into Two equal Parallelograms.

Two assistants are to take hold, one at each end, of the cord of one hundred yards, which, as already stated, is

divided into ten equal parts of ten yards, with appropriate marks of distinction. The leader is to take the small sticks in his spare hand, and, leaving his follower at the point O, is to move on, until the line is straightened; when the follower calling aloud, *one*, the leader is to stop, and fix one stick at the end of the cord; he then carries the line on, the follower proceeding after him, until the hind end of the line in his hand comes to the place where the leader had fixed the stick; on which he calls *two*; the leader, on this, again stops, and fixes a *second* stick, after which he proceeds as before, the follower having taken up the *first* stick. In this way the whole side is measured, the number of sticks set down giving the number of lines, (each 100 yards); the knots between the point P, and the last stick fixed by the leader, giving tens of yards, and the number of odd yards and feet being ascertained by applying the ten feet pole. Now, if the side OP measures four hundred and seven yards, there must be measured back two hundred and three yards, and one half, which will give the half of its whole length, and consequently determine H the middle of the field; for it is plain, that a perpendicular bisecting the line OP, must divide the field into two equal and similar parallelograms.

To describe this perpendicular, let the standard triangle be laid on the ground, the bottom spike pointing to either O or P, (it being indifferent in this instance); say that it be towards O, and let the inner corner, G, of the triangle, touch the point of bisection. Then the surveyor, taking the whip-cord, and keeping it tight, (by raising his hand above his head), should look with one eye (the other being closed), along it, so as to fall in with one of the wands which should have been left at O, and causing the inner edge, GF, of the standard, (that is to say what would be his plumb-line, if the standard were set at its point), to coincide with the sight he has along

the whip-cord; by this means the standard must point truly towards the point O, and the rectangle, CA, of the triangle, must give another direction, perfectly perpendicular to the line OP, in the field.

The surveyor now shifts round a little to his left, and holding the whip-cord as before, but directing it along the line GE of the triangle, with which it is to coincide, ascertains the exact place, T, in the side of the field, NS, where it will be bisected.

During the time of taking the first sight, towards O, one of the assistants should proceed to the middle of the side NS, to be ready to fix a wand at the ascertained spot. If, in the first instance, he should be a little too much to the right or to the left, the surveyor would either call, or make a signal with a wand, for the assistant's moving to the proper situation at T, which being done with due precision, the surveyor should, in a significant manner, fix the wand in his *own* hand into the ground, as a signal to the assistant that *his* position is correct, and for him to fix the wand he has with him. A line drawn from H to T will divide the field as required.

EXAMPLE II.

I SHALL now explain the proper mode of cutting off a triangle from a field, it being an operation of great moment, and, in fact, the basis of practical surveying. A very little observation will instruct as to the proper manner of dividing a field into triangles, parallelograms, or trapezia, so as to work with most readiness.

The field H G M K is irregular, in having the side KM longer, by forty-six feet, than the side HG, which consequently occasions HK to be longer than GM; there-

fore, leaving a wand and flag at K, we proceed to find the spot, L, (indicated by the standard triangle, placed on the ground, with the spike pointing to K), whence a line drawn from the point H, would be perpendicular to K M. A wand and flag should be held by an assistant for that purpose at H, to give the direction to the surveyor's whipcord. As before stated, the triangle must be moved either to the right or left, as the surveyor may find by his whip-cord he cuts to the right or left of H. When the line, the wand at H, and the sight E G on the triangle, all coincide, and that the side A B of the triangle points correctly to K, then the corner A of the triangle will give the spot L, where the third flag should be fixed, and from which the assistants should measure the distances to K and to H.

Those sides will afford the following measurement:

The base line, K L, is 46 yards:

The perpendicular, H L, is 79 yards.

The hypotheneuse does not, in this instance, require to be measured; but to bring the contents of the triangle K L H to a parallelogram, only half of either of the above measurements is to be multiplied by the whole of the other; that is, either 79 may be multiplied by 26, or 46 by $39\frac{1}{2}$; the former is easiest, as it avoids the fraction.

This calculation is founded on the rule, (vide Problem XXIV.) where it will be seen, that a triangle is equal to half that parallelogram, whose base and altitude are equal to that of the given triangle; therefore,

Take 79 yards multiplied by 23 = 1817 square yards.

	79	
	23	

	237	
	158	

		R. P. Sq. Yds.
Divide by square yards in a rood, 1210)	1817 (	1 20 2
	1210	

	607	
Reduce to $\frac{1}{4}$ of square yards . . .	4	

Divide by $\frac{1}{4}$ of sq. yards in a pole, 121)	2428 (	20
	242	

	4) 8	

Sq. Yds.	<u>2</u>	

We have the triangle K L H separated and measured, the area being, as above stated, one rood, twenty poles, and two square yards.

With regard to the residue of the field, it being a rectangle, is easily measured; its length, 134 yards, being multiplied by the perpendicular, 79 yards, will give the contents of its area, as follows:

$$\begin{array}{r}
 134 \\
 79 \\
 \hline
 1206 \\
 938 \\
 \hline
 \text{Square yards in an acre, } 4840 \text{) } 10586 \text{ (} 2 \text{ } 0 \text{ } 29 \text{ } 28\frac{3}{4} \\
 9680
 \end{array}$$

$$\begin{array}{r}
 \text{Square yards in a rood, } 1210 \text{) } 906 \text{ (} 0 \\
 \text{Reduce to } \frac{1}{4} \text{ of square yards } \cdot \cdot \cdot 4
 \end{array}$$

$$\begin{array}{r}
 \frac{1}{4} \text{ of square yards in a pole, } 121 \text{) } 3624 \text{ (} 29 \\
 242 \\
 \hline
 1204 \\
 1089 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Restore to square yards } \cdot \cdot \cdot 4 \text{) } 115 \\
 \hline
 28\frac{3}{4} \\
 \hline
 \hline
 \end{array}$$

	A.	R.	P.	Sq. Yds.
Total above	2	0	29	$28\frac{3}{4}$
Add the triangle	-	1	20	2
Makes the field to be in the total	2	2	10	$-\frac{3}{4}$

N. B. As the pole contains a fraction, *i. e.* a quarter of a square yard, it is proper to reduce the whole pole to quarters of yards, before using it as a divisor; by dividing the residue by 4, it restores the fraction to integers, except any quarters that may remain, which cannot be more than three.

It is also to be remarked, that the field, of which we have just treated, being of a regular breadth throughout, it might have been more readily measured by bisecting the line LH in O, and thence drawing a mean diameter to P, in the oblique side, by which only one calculation would have been necessary; it would have been 79 multiplied by 157, or vice versa.

EXAMPLE III.

THE learner has been shewn how to calculate a square, a parallelogram, and a right-angled triangle; the following lesson will instruct him how to ascertain the contents of the obtuse-angled triangle, EFG.

The surveyor should measure from G, leaving a wand and flag there, and when arrived at H, should take a back sight to G, as before directed, taking care that the standard lays in an exact line with the whip-cord, and that the whipcord coincides with the wand at G; which should always be placed as nearly perpendicular as possible.

An assistant should proceed to the corner, F, and there fix a wand, for the surveyor to ascertain whether or not he is arrived at that spot which makes FHG a right angle. When that point is established, a wand is to be fixed at H, the measure thereto, 160 yards, being recorded, that the altitude may be rightly stated, when drawing the field on paper; the assistants are now to proceed, and finish the measure up to E, which gives 75 yards more, making the whole line 235 yards. Going back to H, the altitude HF is to be measured, which is found to be 68 yards. The length of the base, 235, being halved, gives $117\frac{1}{2}$; which is to be

EXAMPLE IV.

THE trapezium, or irregular four-sided figure, next claims attention. This occurs very frequently, but is constantly divided into two triangles, each of which is measured according to the foregoing rules.

A familiar acquaintance with mathematical figures will soon enable the student to decide, at first view, how a field should be divided for measurement. The shortest way is, to draw the first line between the two most distant corners of a trapezium, as G K, by which means the process will be shorter than by drawing from I to H. In the latter instance, it would be requisite to make three measurements, viz. one from I to H, the second from G to H; and both those triangles must have perpendiculars to ascertain their altitudes, as usual. By taking our diagonal from G to K, we are certain of having the longest side of each triangle made thereby, and the offsets, or perpendiculars O I and S H are all that are wanted to render the measurement complete, which will appear from the following calculation:

Say,—From K to G is 394 yards.

From S to H is 113

From O to I is 197

Now, as both triangles have the same base, we have but to add the length of the two offsets, viz. 113 to 197 yards, making a total of 310 yards, and the half thereof being multiplied by the base, will give the contents of the area. This precludes the necessity of calculating each triangle separately.

Base is 394

Half the two offsets is 105

1970

394

A. R. P. Sq. Yds.

4840) 41370 (8 2 7 18 $\frac{1}{2}$

38720

1210) 2650 (2

2420

230

4

121) 920 (7

847

4) 73

18 $\frac{1}{2}$

I hope that, hitherto, every thing has appeared perfectly intelligible and easy in practice; if the least doubt should exist, it is absolutely necessary for the student to remove it, before he proceeds any farther; for it is not possible he can comprehend what follows, if any uncertainty is left as to the past; not that any great difficulties are at hand; but, it must be evident, that the foregoing must be completely understood, before any attempt is made to proceed with that superstruction, of which it is the foundation.

EXAMPLE V.

Is an irregular Pentagon, or Figure, having Five Sides.

DRAW the line I K, and set off the perpendiculars N H, O L, and M G, which will divide the pentagon into three triangles and a trapezium.

The following will be the measurements: say, that,
 N K 67 yards, N H, 57 yards, make the triangle, N° 1.
 I M 46 G M, 48 make the triangle, N° 2.
 I K 168 O L, 34 make the triangle, N° 3.
 M N 56, the sides 48 and 57, give a mean of $52\frac{1}{2}$ for N° 4.

Multiply the whole length of each of the three triangles, by their altitudes, thus:

N° 1.	N° 2.	N° 3.	The Trapezium.
67	46	168	56
57	48	34	$52\frac{1}{2}$
469	268	672	28
335	184	504	112
			280
3819	2108	5712	2940

Add their products.

N° 1 3819

N° 2 2108

N° 3 5712

Halve it . . . 2) 11639

$5319\frac{1}{2}$

Add the trapezium . 2940

Total area sq. yds. . $8259\frac{1}{2}$

	A.	R.	P.	Sq.	Yd.
4840) 8259 $\frac{1}{2}$ (	1	2	33	$\frac{3}{4}$	
					4840
1210) 3419 (	2				
					2420
					999
					4
121) 3996 (	33				
					363
					366
					363
					3, or $\frac{3}{4}$

The reason for multiplying the whole lengths of the several triangles by their whole breadths, is, that there is less trouble in halving their aggregate product, than in halving one side of each ; in the last method we are subject to many fractions, which are avoided by taking them according to the mode just practised.

The two sides, G M, H N, of the trapezium, are of unequal lengths, but they being added together, and their product being halved, gives a mean altitude; that is to say, the altitude of a perpendicular, D Z, Fig. 2, at the center of its base; for we must consider the upper part as being a triangle, crossing at the head of a parallelogram of equal base, and, as the diameter of a triangle at its center, multiplied by its base, gives its whole contents, so, in this instance, we find Q D to be a mean between the whole diameter, or altitude, L K, and the other end, H, which has not any diameter, it being merely a point.

As this often occurs in surveying grounds, in which the hedges or boundaries indent, in various shapes, the

student should, on all occasions, pay attention to forming trapezia, where his base, or diagonal line is long, and admits of his including two figures under a mean calculation; but it should only be done where, as above shewn, the sides are perpendicular to the base-line, and consequently parallel between themselves.

EXAMPLE VI.

It sometimes happens that mounds, walls, heavy masses of wood, &c. intervene, and prevent the surveyor from taking advantage of favourable angles, in which case he must divide his field in the best manner such impediments may permit. I now give an instance where the principal lines of division are obstructed by the above causes.

If the house, V, were out in the center, the division of this irregular figure would be remarkably easy; the following triangles would then take place:

KLP, LON, GVL, HIV.

As it is, the work must be GQSL, trapezium; LSTN, trapezium; HQRI, trapezium; KIOP, trapezium; and IRT O, trapezium; the house, V, separately measured as a parallelogram.

By this means advantage is taken of fixed points, which are, on all occasions, extremely desirable, and the edifice on the mound between L and O is avoided.

Having given the student so many instances of calculation, I shall not burthen the work with what would be requisite to solve this example; besides, it is time that he should begin to exercise himself in managing the demonstrations; therefore, I confine myself to pointing out the manner in which such a plot ought to be divided for survey.

In complex surveys, that is, all such as have many divisions, I have been in the habit of using two sets of wands, viz. some with red, and some with white flags, applying one set to the primary divisions, into trapezia &c. and the other to the offsets, from the base-lines of triangles; being light, they cannot be considered as an incumbrance, while their obvious distinctness prevents confusion.

EXAMPLE VII.

WE are occasionally so situated, as not to be able to follow our survey at pleasure, on account of woods, morasses, buildings, or other obstacles. In such case, we must work our way round, as will be seen more fully described in Example X. As a suitable preparation for that lesson; I shall now instruct the learner how to trace any figure by its circumference; so that a line, which cannot be measured, owing to any of the foregoing causes, may be determined both as to its length and as to its direction.

The figure, $ABCD$, represents a trapezium, of which three sides can be ascertained; but the fourth, being intersected by a morass, is inaccessible. We must, on this occasion, establish the exact situations of the points A and D , whence we can compute the line by which they are separated, and which they terminate.

Draw BE , rather inclining to C , than perpendicular to CD , so that an offset may be taken from some part, F , at right angles with BC , which shall point to D . From the other end of the line BE , carry an offset to A , in the same manner from G .

It must be particularly noticed, that F becomes the primary station, since it is by measuring the lines FD and FE , that we find the length of the hypotenuse,

DE; which being prolonged, according to measurement, reaches to C; then, by prolonging the line EF, by ad-measurement, to B, we find the point where the line CB terminates, of which the point C was ascertained before; consequently, we have all the triangle EBC perfectly correct. It remains to fix the point A, which is easily done; for the line GA being perpendicular to EB, and being measured, we know where the line B must meet the line AG; therefore, as we have three sides established, the fourth line is found, it being an axiom, that in all polygons where all the sides except one are given and fixed, the remaining side is, in effect, given also.

The student will see that all this was derived from the station at F, and that no difficulty exists of tracing lines round any object, provided right angles can be found either within or without the limits of the given figure. To prove the latter position, I will tender a brief illustration in Fig. 2. Having no access within the angle ABC, let the situation of A be ascertained by exterior survey.

Make an offset from C, at pleasure, to D, and draw the line DE, touching the point B; then, from E, make an offset to A. All these lines, being measured, cannot fail to establish the point sought; for the triangle CDB gives the direction of BC; and EA, being drawn perpendicular to ED, establishes the place of A, by its falling upon the line BA in the point A.

The student will perceive, that wherever he has an opportunity of constructing a triangle of any kind, however small, provided the measurements of its sides can be faithfully taken, he has the means of following the course of a line, and to pursue it through all its angular deviations. He has been shewn, in Prob. XV. which teaches how to construct a triangle with three given lines, that they can form but one shape, as is proved by cutting out that figure

on a piece of paper; for, turn it whichever way, the same angles appear, joining the same sides; they are truly immutable: hence it appears, that whenever a third line is added to two in a figure, we have only to ascertain where that third line crosses or intersects either of the others in its prolongation, to establish a knowledge of its direction; and if its length be then measured, the point to which it was directed is also fully established.

Let us revert to the figure just described, and continue a third line, which shall reach the point F. If we continue the line EA (of which the direction is known) to G, and thence carry a perpendicular offset, of which we take the measure to F, the line AG being before measured, and the angle AGF being known to be a right angle, we have two sides of the triangle, of which AF is the hypotenuse, and whose measurement cannot be mistaken.

Thus we see, that although the interior of a figure should not be accessible, we cannot be at a loss to find any desired spot, provided a small space is given for our operations; but where that is not allowed, the standard triangle will enable us to overcome every such difficulty, as will be evident when the use of its quadrant is understood.

I have judged it expedient to instruct the pupil in the application of triangles, as being of the utmost importance, where a quadrant, or other gradated instrument, cannot be had; hence he will not feel himself at a loss, as many persons reputed to be extremely expert, have done, when their costly instruments have not been at hand.

EXAMPLE VIII.

THE irregularity of inclosures presents to the learner a most useful lesson, as well as a trial of patience. Sometimes we find boundaries so very capricious in their wind-

ings, as to cause our wondering at the ingenuity of those who planned them! I shall, in this, give an exact representation of one not many miles from London.

By means of the base-line, A H, the manner of dividing such a boundary, for the measurement of its several parts, is rendered very easy; in fact, the whole becomes a series of triangles of which, all but one, viz. B C D, touch the base-line. There is a curve from D to O, which is not of any disadvantage; for the trifle gained within the curve, is counterbalanced by the loss near to O. Nor, on such an occasion, should we hesitate at the small difference arising at F, where the line stands a little apart from the boundary, and leaves the triangle O E F rather imperfect.

The curve at the side C B of the space C B D, is cut off, and an offset given from the chord C B: if two thirds of that offset be taken for the altitude of a parallelogram, of which the chord is the base, it will go very near to the actual measurement of the segment thus divided, and leave D C B a right-lined triangle.

Between F and G we have another protuberance, but less regular in its form; therefore, we make three offsets, as may be seen by taking the following, Fig. 2, for explanation, on a larger scale.

The learner will see how even the most irregular limits may, by the means of offsets, be brought into mensuration with sufficient precision.

Let the irregular line, A, B, C, D, represent the hedge, or boundary of a field, on such a scale as should render it ineligible to examine it very closely. In proceeding from A to D, in a straight line, take offsets to the several prominent parts of the boundary, noting down on what part of the base-line, A D, they are situated, and recording the length of each offset, from the base-line to the boundary.

To ascertain the contents, suppose A to be one of the points of calculation, but there being no height, we count only the line F, which is 6 yards high, and cuts off 10 yards of the base; therefore, 6 multiplied by 10, gives 60. We next take the space, viz. 8 yards from F to G, and we find G is 5 yards, which added to the other extent at F, viz. 6, gives 11; that multiplied by the quantity of base between them, viz. 8 yards, makes the product 88. Next we find 6 yards between G and H; and as the latter is 8, and G was 5 yards in length, the two added, give 13 yards to be multiplied by 5, the quantity of base between them; they give 65 yards. We next go from H to K, 4 yards; K being 7 yards, and H being 8, they make 15, which multiplied by 4, their immediate base, gives 60: then, from K to D, we have 11 yards, which being multiplied by 7 (the height of K) gives 77.

The following is the aggregate of the calculations:

From A to f, 60 square yards.

From f to g, 88 D°

From g to h, 65 D°

From h to k, 60 D°

From k to D, 77 D°

Total - 350, which sum 'of 350 being halved, gives 175 square yards for the superficies, = about 5 poles and a half.

EXAMPLE IX.

THE foregoing examples are founded on the supposition, that no water intervenes, and that the whole is, except in the last instance, a level plain. I now offer an

example, where both wood and water intervene, and require a different treatment from any thing contained in the preceding pages. The student will observe, that the operations required for setting off perpendiculars to the borders of a lake, or a wood, apply equally to finding the contents of small irregularities, or curves, in the sides of fields, &c.

For the reason assigned in the illustration of the last example, I shall forbear from doing any thing towards the calculation of such parts of this figure, as have been already, in my opinion, sufficiently explained; confining myself to the detail of such minutiae as depend on the novel circumstances now introduced to the student's notice. I am to conclude he is acquainted with the treatment of trapezia, and of triangles, and that nothing more than a little exercise in that part of the study is required to give a due confidence, and lead him boldly to that readiness in practice, without which his efforts will be of little avail. The truth is, that the various figures occurring in this fascinating art, in a very short time create an intimate acquaintance with their proportions, and render objects of seeming difficulty so familiar, as to be comprehended fully, without the smallest hesitation, after being once walked over.

The tract GIKLH contains the wood Y, and the piece of water, XXX, which intersect all the right lines that could be drawn between any two points subtending an angle; for, neither from L to I, from K to G, from G to L, from H to K, nor from H to I, could a line be drawn, without obstruction; we are therefore compelled to seek for intermediate stations, where, fixing a flag, we obtain an artificial angle, formed by the junction of two or more lines, during the progress of the operation. Such stations as that at S, are commonly known by a small circle with a dot in its center, thus $\odot$.

The advantage of the position at S must strike at first view! for it immediately gives the two triangles, G S I and I S K, and, by means of the two offsets to P and Q, affords the triangles V P B and Q K W; again, another offset is made from Q to R, which leads on to N; and, as the border of the lake is so nearly straight from N to T, we have the pentagon, Y, formed by the angles at R, N, T, L, W. The small projection, Z M N, may be treated as a triangle, of which M is the summit.

On the other side of the lake, the offset at P being at right angles, and the line of the perpendicular from P going direct to H, we have the hypotenuse G H, and the two small triangles, B V P, and B C S, come easily under our management.

From the point V, in the line P H, we have a projection of land; to measure which, we take the three offsets, from 1 to 2, from 3 to 4, and from 5 to 6. To estimate the contents of the projection, let the center line, 3, 4, be taken for the height, and 1, 5, for the base of a parallelogram; then, from V to 2, for the base of a triangle, and 1, 2, for its height; and the same on the other side, 5, H being the base, and 5, 6, the height of a second triangle. Multiply the numbers of the whole sides of those triangles, and taking half their product, add to the amount of the parallelogram, 1, 5, 2, 6, which will be sufficiently near to the real area of the projection. Observe, that something is gained in the computation of the parallelogram, which is again lost in the triangles: on a small scale, this cannot prove any difficulty; and, on a larger one, there will be found means of making various figures, so as to leave only a few ragged borders, or remnants, which may be thrown alternately into either scale, without creating any sensible difference. It is true, that we have means to ascertain all such matters, however small; but either the modes of computation, or the instruments,

are such as do not come within the limits of our present intention.

We are now come to the wood Y, included in the pentagon RNTLW. The direction of the side WL is known by its being a continuation of the line KW; it remains to measure WL; but the wood Y, debars us from seeing the direction of the line TL, which is part of the line HL; we must therefore form the small triangle RDZ, making the line ZD perpendicular to the line RW, of which it is an offset. This triangle gives us the direction of the line RN; for, having the measure of two sides, RD and ZD, the length, and, consequently the direction of the line RZ, are ascertained. Having thus found the angle at which RN stands from RW, we take its measure; thus we have three sides of the pentagon.

Again, we form the triangle ZMN rectangular at M; and, as the line MT is nearly straight, we take its measure. Now, MZD being in one line, and ZMN being a right angle, the line MT must be parallel to RW; therefore, the length of NT being laid down, will intersect and shew the length of TL, which could not be ascertained without this circuitous mode of examination.

OF THE CONSTRUCTION AND USE OF THE QUADRANT, ON THE STANDARD TRIANGLE.

THE formation of the three limbs of the triangle, has been already, I trust, so fully described, as to require no further notice; but it remains to instruct the student how to gradate the scale of 90° on the hypotenuse. Take a piece of very thick white paper, and describe thereon the fourth of a circle; the larger the better; then, setting the center of your protractor to the center from which the quadrant was drawn, and seeing that the edge of the pro-

tractor fits to one of the lines; as in Fig. 1, set off all the degrees from its gradated edge, by small punctures for every five degrees, and simple dots for every ten degrees. Fixing your paper firmly down to the table, by means of weights, &c. drive a short thick pin into it at the center, and with a silken line, looped, and kept down to the bottom of the pin, so that the strain may not bend it, go regularly, by gradations of 5° , marking them down on the quadrant you had drawn; see that they are all correct, and number them, thus; 5, 10, 15, &c.; after which, you must draw your silk through each single degree, in the same manner, marking them in between the former divisions of five degrees, so that every one of the 90° will be traced to its proper place.

Observe that you select a fine length of sewing-silk for this purpose; if it be waxed, so as to make it lay in a smaller compass, and take off all irregularities from its surface, all the better. Your paper should, in preference, be triangular, exceeding the size of your work every way, one or two inches, and should now be pasted neatly to your triangle, the center being placed *exactly* at the right angle, where the plumb is to be suspended; for that precise spot should afterwards be occupied by the center of the stud, which is screwed to the corner of the standard. Having pasted the paper neatly to the frame, so as to make it sit firm and smooth, let it dry gradually; and, when dry, sponge the paper on which the work was done, moderately; then leave it to dry again, but do not sponge the part that was pasted to the wood. By this process your paper will become as tight as a board.

Now, screw on the stud, as above directed, and looping on a fine silken line, carry it out, so as to include the wooden stay, or hypotenuse, of your triangle, which should be a very clean piece of fir, though box is preferable. Set off every 10° as before, upon the wood, by

a line made with the edge of an inch chisel, precisely in the track of your silk. Set off all the five degrees in the same way, with an half-inch chisel.

The intermediate degrees are to be marked with a quarter-inch chisel. All these marks are to be very accurate and distinct.

This being established, let the marks be all burnt in with the same instruments, heated rather moderately; that is, just sufficiently to stain the wood permanently, without singeing or chasing its edges.

Fig. 2 shews the paper pasted to the frame: the spaces beyond the gradated part should not be cut away, unless they wrinkle up, and obstruct the line, as the paper might be apt to tear at the edges, creating much trouble, and some incorrectness.

The same paper, if carefully managed, will serve to gradate both sides of the triangle, otherwise that may be done by means of a bevil.

With a standard triangle, thus completed, every kind of survey, occurring under common circumstances, however extensive and intricate, may be taken. By means of such an instrument, we are enabled to take angles either horizontally or vertically, at pleasure; and thus an infinity of circuitous trouble may be saved.

The student should be extremely careful that his instrument be not placed in a leaning position against a wall, for its own weight will give it a cast which must derange the accuracy of every survey taken with it in such state. The standard should always be hung up, when not in immediate use, in the manner we hang up guns, &c. by two nails, or hooks, one at each end, the angular part of the instrument hanging downwards.

The plumb-line, as well as the sight-line, should be neat, and of an even surface; the plumb itself should be in shape of a bell, pierced exactly through the center, so

as to hang even; its diameter at bottom to be about one inch, and at the top just broad enough to touch the edge of the standard, according to the annexed figures, which will, at once, demonstrate when the instrument is perpendicular.

The quantity of wood scooped out, ought to be of the same shape with the plumb, and in the angle of the standard, not so as to reach through its thickness; for the corner affords the means of estimating the uprightness of the standard in both directions, as may be seen in Fig. 4, A being the center of a cross formed by the lines of the front, and side-lines of the wood.

Hitherto, the learner has been taught to proceed by recourse to a succession of triangles, especially in the latter examples; I shall now shew a shorter way of ascertaining angles with the standard triangle, observing, that when the ground may be very uneven, the instrument should be aided to a due inclination towards the object to be intersected, by placing clods, turf, &c. under the deficient side; thus, on level lands, the standard should be brought tolerably to a level, as in Fig. 5; but, if the surveyor be on a hill, and would take a sight towards an object in a vale, or, *vice versa*, the standard should be inclined in the direction thereof, as described in Fig. 6.

EXAMPLE X.

THE corners of fields, &c. are not always rectangular; indeed, except in grounds laid out on a large scale, or inclosed from commons, we rarely find any kind of regularity. This necessarily occasions an immense deal more of labour in surveying them, than would be the case if the land were partitioned into squares and pa-

rallelograms; we must endeavour to oppose the evil by some of the many means with which mathematics come to our aid.

The standard triangle is capable of measuring, as well as setting off any angle; the operation is extremely simple, and, with a little care, perfectly certain. Suppose it were required to measure the angles of a figure, in which both acute and obtuse angles are to be found, and that, without taking any sight from one point to another, across the field, &c. it is required to lay down the whole upon paper, in fact, as though it were a polygonal lake, to whose interior we could not get access.

First, to measure an acute saliant angle, that is, one which projects, such as resembles the wedge.

Let B, A, C, be the saliant angle, which is to be measured exteriorly; let the standard, DA, be placed so as to be precisely in a line with AC, its corner being applied exactly to A, so that a sight taken from F, may look down the line AB; when the sight is taken, by means of the whip-cord, let it be laid carefully down, so as to touch that point on the graduated hypothenuse, which indicates the value of the angle. If the angle BAC be 60° , the angle DAF must be the same; for when two right lines intersect each other, the opposite angles will be similar.

EXAMPLE XI.

To illustrate the foregoing case, let the instrument be laid down in the opposite direction, as at IRS; and the line being laid towards G, so as to correspond exactly with NR, a stick is fixed at the spot Q, to mark where it pointed; and the instrument being swung round, so that the point Q should come exactly to R, and S

towards G, the plumb-line being carried to the stick at Q, will cut the hypotenuse at that point which will shew the measurement of the angle, reckoning from the upper point I of the hypotenuse.

Observe, on all such occasions the stud and inner parts of the triangle are to govern; for instance, in measuring the angle just described, the outer edge of the frame, L, should be pushed beyond the angular point, R, until the inner corner of the triangle, O, corresponds with the end of the wall, &c.; in other words, until the four points, N, R, O, G, are all in a right line.

The point O, where the stud is fixed, (from which the scale is drawn), and the point Q, which is the true perpendicular to OS, are to be the guides. The student cannot be at a loss as to this matter, when he recollects that the line can never measure any angle on the upper limb, but only on the hypotenuse; consequently, that all angles are to be taken between the space IZ, of all which O is the common center.

EXAMPLE XII.

A DIFFERENT position of the standard is requisite when an obtuse saliant angle is to be measured. In this operation, the standard is to be kept more within the point than in the preceding instance; but the principle is precisely the same, for the points XZO must be all in one right line.

Here the surveyor at once decides the value of the angle, for by laying his line on the hypotenuse, he finds that the number of degrees cut off, on his right hand, between N and M, is the sum in which the angle is short of being 180° ; thus, if 26° be between N and M,

the angle ZXQ , or, (which is the same) ZOL , must measure 154° ; or, he may add 90° to the quantity between M and L , viz. 64° , which will come to the same amount.

EXAMPLE XIII.

WE have now to measure a re-entering or retiring angle, that is, in form of a funnel; this is extremely easy, for it does not require any sight to be taken, the work being effected by first laying the standard one way, as at ABC , and reversing it the other way, as at IDB ; the number of degrees overlapped, by crossing of the two directions, indicate what the angle is short of 180° , as is shewn in the following figure.

Bring the first point, B , of the standard, ABC , to the angle, O , in the wall, FOQ ; let an assistant keep your plumb-line, or any piece of thin twine, stretched from B through the point A , which is the inner line of the upper limb of the triangle; then reverse the standard, and, as before, bring the corner, B , to O . The line held by the assistant, will cut the hypotenuse at G , from which, to H , contains the number of degrees wanting to complete 180 ; thus, if there be 43° between G and H , that sum deducted from 180 , leaves 137 , the value of the angle; or, add the number of degrees between G and X , to 90 , and they will give the same amount.

N. B. The surveyor will not fail, when he reverses the standard, to draw the whip-cord through the perforation in the iron plate opposite the stud, as that will now become the central point, whence the angles taken with the reversed instrument are to be measured.

EXAMPLE XIV.

It remains to shew how an *acute* re-entering, or retiring angle is to be measured; it is evident that, in such, the standard cannot be carried up close to the angular point, therefore the operation must be brought out a little way.

Measure off, from O, equal distances to A and B, set down the standard at A, and draw the line AD; remove the standard to B, reverse it, and draw the line BE; the intersection of these lines, at C, will give two angles, BCD, and ACE, each equal to AOB, which may be measured with the hypotenuse, placing the point at D, (or E), and bringing the stud to C; or these angles may be ascertained by measuring the angle DCE, the value of which being deducted from 180° , will leave the residue for the measure of the similar triangles, DCB, or ECA.

We have also another method of measuring retiring angles, where the sides are too short to admit of the foregoing device; it is a quick, and easy way. From the point A, of the angle BAC, (Fig. 2) measure off two equal distances along the walls AB and AC, by means of a line to be held by a person at A; two others are to hold the other ends of the lines, as also a third line, reaching from B to C, keeping the whole tight; they are to apply the triangle, as in Fig. 3. The lines are to be carefully preserved at a moderate degree of tension; the person at A to apply the part he holds to the stud D, on the standard, the other two retaining their measured distances, and applying their line thus; the line AB to go along DE, and the line CA to go along DF, which latter line will shew on the hypotenuse what angle is made by the walls.

Where an angle is so very acute as not to admit a person to go close in, to hold at the point A, the line should be put into a notch at the end of one of the wands, and be therewith thrust into the corner. If the cord be well managed, and be not too thick, it will answer well enough; in particular, good whip-cord; but pieces of wood, such as battens, wands, &c. are, generally speaking, preferable.

Having now carried the student through the several *prostrate* uses of the STANDARD TRIANGLE, and, I hope, rendered every part of each operation perfectly clear, I shall proceed to instruct him in what relates to surveying, with the STANDARD, in a PERPENDICULAR POSITION. He will find a new scene open to him, replete with pleasure, and answering more purposes than merely the ascertaining of distances, or spaces, as relating to lands.

EXAMPLE XV.

To take a sight of an object standing at a distance, on a level with the standard triangle; let the instrument be placed perfectly upright, so that the plumb may rest in its proper place, neither inclining one or other way. When this is effected, the top of the triangle will give a true horizontal line; it being then at right angles with the plumb-line, which must ever be a vertical perpendicular.

This position is expressly applicable to what is termed "taking levels;" by which it is either meant to find out the levels of particular objects, or to fix on such as correspond, by their height, with the true level of the instrument; that is, with the limb, or line, that guides the eye to the object in view, *i.e.* E D in the annexed Fig. 3.

For this purpose a groove should be made in the upper limb of the standard triangle, about a quarter of an inch

deep, and of the same width; whereby the sight will be more particularly confined, and enable the surveyor to take the exact altitude of the object. This groove should be remarkably smooth, and free from fibrous raggedness; which would intercept the line of sight, and prove very unpleasant. The plumb-line should also be separated from the whip-cord, by the latter being passed through the hole in the plate on the reverse side; by this means there will be less danger of deranging the plumb-line, which should, on all occasions, be kept as steady as possible; since on its true position will the whole of the estimate entirely depend.

Say, that it is required to find the difference of level between A and B, the former being situate on a plain, the latter on a gently rising hill. Ex. XV.

Let the standard be placed in the direction of B, perfectly plumb, and let an assistant proceed to the spot B, having with him a staff of about 12 feet long, and an inch and a half, or more, in diameter; on this staff, a target, either square, or circular, of a foot in diameter, is to slide up and down, rather stiffly, so as not to move but when pushed. The face of the target should be painted in three stripes, two of black, and one between them, of white; the black stripes may be about 5 inches each in depth, but the white one should not, at farthest, be more than half that breadth; indeed, the narrower the better for this kind of work; as the levelling will be more correct.

The surveyor, looking along the upper line of his standard triangle, makes signals, by raising or lowering his hand, to the assistant at B, according as the target is to be more raised or lowered; and, when the white line across its center appears exactly to coincide with the level of the instrument, he then quits his position, and proceeds to examine the number of feet and inches at which

the brace, or ring, that keeps the target to the pole, (and which should be exactly opposite the back of the white line's center), stands from the ground at B. Now, if the part D of the horizontal line of the triangle be five feet three inches above the ground, which is allowing three inches of spike to be buried therein; and, that the ring indicates three feet, seven inches, to be the height of the white line above the ground, at the spot where the target is held by the assistant, deduct 3 feet, 7 inches, from 5 feet, 3 inches, and the remainder, viz. 1 foot, 8 inches, will be the height of B above the level of A.

This mode serves sufficiently to ascertain matters that are not very nice in their nature, such as the angle at which a road ascends a hill; or, to ascertain the highest part of a field to be built on; or to make drains, &c. where particular accuracy may not be indispensable; but, where measurements become *minute*, let two pieces of thin wire be drawn tightly across over the groove, one at each end; they may be fastened to nails, buried up to their heads in the sides of the horizontal piece; these will serve as guides; and, by being both brought into one line, (the surveyor standing a little back from the instrument, as he takes the sight), and cutting the very center of the white line on the target, precision will be nearly perfected. A piece of iron in form of a T at each end, will be preferable, where vertical lines are to be cut, as it will answer the purpose of the cross bars in a theodolite; but great care must be taken that the upright line be exactly central in the groove.

Let us now reverse the operation, and take the sight from the more elevated spot, to find the level of that on which the instrument stands; this, generally speaking, is an object of more importance than the other, although the two play, as it were, into each other's hands, and are taking the lead of each other constantly. The reason of

its superior importance in the average of practice is, that we rarely want to lower water, though we seek every opportunity of preserving its height, or of returning it to the level of its source; nay, eventually, higher!

One circumstance must always be carried in mind while levelling, namely, that the earth is not flat, but, that every line we can measure on its surface, though apparently level, is part of an immense circle, drawn with a radius of *four thousand miles!* that is, with half the earth's diameter, which is 8000 miles. This occasions our not perceiving the curve we describe in a long walk, but which, however, exists, not only in the imagination, but in reality. We find, from calculation, that the difference between the apparent and the true level is, within a very small fraction, eight inches in a mile, or one inch in every furlong; and practice confirms this estimate; for we are obliged, in all aqueducts, to allow nearly half an inch in each hundred yards, as we proceed. Thus, if we are to establish a canal between two places fifty miles asunder, we should, in the course of the work, have to depress it no less than four hundred inches, equal to thirty-six feet, four inches; and this, whether the work began at either end.

To such as have never been informed on this point, or who never considered the fact, what I have asserted must appear paradoxical; it is, nevertheless, strictly true, that, but for the described caution, every apparently level piece of water would have a declivity, of eight inches, in every mile of its length, and must, consequently, become a rapid stream; indeed, a perfect torrent; for, the descent of one inch in that distance, creates a very perceptible current.

For the mode of applying the T iron at each end of the groove in the upper limb of the standard triangle, so

as to cut objects either vertically or horizontally, see Fig. 4. Plate 18.

For the shape of the iron, which fastens on with two screws, see Fig. 5.

EXAMPLE XVI.

IN this instance the target is below, and the sight is taken from above. The assistant should be provided with a forked stick, of about six feet long, whereby he may with one hand push the target up to the top of the pole, if necessary, steadying the pole with the other hand. The sight is taken as before. Supposing the brace, or ring, to stand at nine feet, four inches from A, the surveyor must deduct five feet, three inches, therefrom, which will leave four feet, one inch, difference between the levels of C and A; that is, the spot whence the sight was taken, will be four feet, and one inch, higher than at A, where the target was upheld.

But, if the distance between A and C be great, the difference between the apparent and the true level must be again deducted; thus, taking the interval to be eleven hundred yards, it would be requisite to deduct five inches and a half; which, taken from the above quantity, viz. four feet, one inch, would leave three feet, seven inches and a half, as the actual difference of height.

Therefore, there might be a depth of three feet, seven inches and a half of water at A; while at C a person would stand dry!

EXAMPLE XVII.

I SHALL, in this, combine the two foregoing operations on a smaller scale, whereby the learner will have a more

comprehensive view of the subject, and be enabled to correct any misconception that may, notwithstanding my endeavour to be clear and perspicuous, have taken place.

A canal, AB, being dug as far as the point B, and it being required to continue it in the same direction, a survey, of course, must be taken, for the purpose of ascertaining the level, whereon the expences must generally be estimated. The hill O intervenes, at the back of which is the hollow, whereof N is the deepest part.

Let the standard be set up at B, and from its top let the sight CD be taken, which will cut the side of the hill at D; remove the standard to the spot, D, from which again take the sight EF; again remove to F; and, having sent an assistant, with the pole and target, to the other side, let him slide the target as high up the hill as may be practicable; gradually getting to that precise depth which may bring the white line opposite to G, so as to give the line GI. Thus far the operation is extremely simple; and, as three successive sights are taken, all of the same height, there cannot be any mistake; for BC is one height of the standard, viz. 5 feet, 3 inches; the level strikes at D, whence another height of the instrument, DE, occurs, giving 5 feet, 3 inches more; the third sight strikes F, whence we have a third height of the standard, namely, FG, making 5 feet, 3 inches more; in all, 15 feet, 9 inches.

Now, G being higher than the top of the hill, H, a sight may be taken to any length of the pole, KI; the more it is sunk, the better, under this reservation, that it should never be fixed on a spot whence a sight could not be taken with the standard triangle; if the declivity be too great, or the spot so stony, that the spike could not enter the ground, the pole should be placed higher up,

or a little to one side, even though it should entail the trouble of an additional station.

From K, a short sight should be taken, just equal to restoring the survey to its level; thus, if the first fall in descending, gave 10 feet, 6 inches, which would exactly counterbalance two sights on the ascending side, the target should be put to 10 feet, 6 inches, from the ground, and brought to be opposite the line of sight on the standard; half the last height would be applicable to the height of the instrument, and the running half would be below the spot occupied thereby; therefore, the foot of the pole would stand at S, on the precise level of AB, and serve as the guide for further operations; for, the level being restored, it is extremely easy to continue it.

The standard should be now fixed at S, where a stake should be driven, to mark the restoration of the level; and, as it is requisite to ascertain how much of the excavation of the hill, O, will be required to fill up the hollow, N, so as to carry the water in a due course, the depth of the hollow, N, must be found, by sending the assistant to one or more stations, and taking the depths. Thus, the pole being at N, the target will be raised to L, where the line of sight will cut it, at 9 feet, 7 inches, from the ground. Take from this, 5 feet, 3 inches, the height of the standard, and it leaves 4 feet, 4 inches, for the depth of the hollow. A substantial stake should be placed at N, either cut to the length of 4 feet, 4 inches, or marked in such manner, as to indicate the depth, under level, at that part. Remarkable places should be thus distinguished, and especially all parts that are, for any distance, wanting of due elevation.

The target should be carried to some spot clear over the hollow, where, being placed at 5 feet, 3 inches from the ground, it may again meet the line of sight: this

may not be always practicable, but, the nearer you can get to the level, the less trouble will be given.

In this manner a canal may be carried to any extent, without danger arising from a want of true level; the process certainly requires attention, as does, indeed, every operation dependant on mathematics.

The standard should be firmly fixed in the ground, and the earth compressed around the spike, by the pressure of your feet; and, if an old spade handle were made into a dibble, and shod with iron, it would, assuredly, prove a convenience in making holes for the standard. On the iron spike, the inches and half-inches should be marked with a file, so that it might be seen, at once, to what depth it held in the ground. This little nicety is not essential on general occasions; but, in levelling for water-conduits, an inch at each station may, in a long course of surveying, make an important difference.

EXAMPLE XVIII.

WE have now to learn the properties of the standard triangle, in surveying lands, from two or more stations, without using either rod or line, for the measurement of distances. This branch of surveying appertains exclusively to plane trigonometry, or the doctrine of right-lined triangles, as applied to even surfaces; that is to say, in contradiction to *Spherical Trigonometry*, which relates to such triangles as have curved lines, and are applicable to spherical or curved surfaces.

I must here remind the reader, that when two sides, forming one angle of a triangle, are given, the other side, and the other two angles are thereby ascertained; and, it is the same in all polygons, where, if all the sides, except one, and its dependant angles be given, the whole figure is ascertained.

Upon this principle the surveyor must often act; but he founds his principal reliance, while surveying with the theodolite, (or any instrument capable of measuring an angle while it takes its sight), on the intersection of two lines, issuing from the respective ends of a base, of which the exact length is known.

The two ends of the base-line are stations, from which sights being directed to any one object, an angle will be formed at their meeting, which angle being imitated on paper, by the means which are the subject of this example, the exact distance of the object will appear. The process is simple, but must be well understood, it being the basis of this branch of surveying.

Take any convenient length of line for your base, as AB , and from each end draw a line to any object, as C ; those lines, AC and BC , will meet at that object; and, as the length of your base is measured, as also the angle each line makes with that base, it is self-evident that the distance of the object is thus made known; for the lines forming the angle mutually intersect; therefore the one determines the length of the other.

The principal beauty of this kind of surveying is, that besides being brief, accurate, and requiring so little labour, it enables us to find the distances, and to fix the positions of objects, which are completely inaccessible.

Before I proceed with the use of the standard triangle, it is proper to submit to the student one or two examples which relate chiefly to marine-surveying, such as taking soundings, following the course of a channel among sands, ascertaining the exact position and distance of a vessel from the shore, &c. by which he will completely understand the principle, before he is called to the practice.

These instances cannot fail to strike his mind very forcibly, and to account for various matters, of which he

can have only a confused understanding; or of which, possibly, he may be totally ignorant; they will also confirm what has been already stated regarding the great utility of the *Triangle*.

EXAMPLE XIX.

A SHIP, being at anchor in a gale of wind, was obliged to cut her cable, and put to sea; previously, however, the captain noted down, that while at anchor, the bearings of different points, or land-marks, were as follow:

The head-land and great oak at A were in the direction laid down, and the steeple of the church, B, and the flag-staff at C, were in one line. This was enough; for, on the gale subsiding, the vessel was brought to the same bearings, and, putting out a boat, soon grappled up the anchor.

What is this but trigonometry? The distances AB, and C, are fixed, and form a triangle, of which one side, BC, being prolonged, directs to the point D, where it is cut by a line from A; therefore, BA, or CA, may be considered as a base, from whose extremities the lines BCD and AD are directed to the object, as described in the preceding example.

EXAMPLE XX.

A SHIP was seen to founder just as it was growing dark; but, as she was going down, the people on shore, at A and at B, took observations as to how the vessel bore; that is to say, in what direction of the compass she was last seen. A boat put off from A, and another from B, each having on board a compass, to direct to the pro-

per point: the boats met at the precise spot where the vessel foundered. Now, either boat, singly, would have, probably, missed of the object, but, by the intersection of their lines of direction, they could not be in error.

Here, again, the distance AB forms the base of a triangle, whose sides meet at C . In this instance the base required no measurement, because each of the lines, AC and BC , had a directing course, which must lead to the point C , common to each line; and, it is clear, that either boat might fall short of, or proceed beyond the spot where the vessel lay, were it not that the other, by crossing its track, shewed the real position of the wreck.

EXAMPLE XXI.

I SHALL adduce one more instance on the same subject.

N. B. The dotted parts denote the situations and extent of the sands, or shoals. The figures imply the depth of water in fathoms; each fathom being six feet.

A boat, being sent to explore the channel of a river, takes soundings, and makes the following report as to the course of the deep water, in the form of instructions to persons navigating there.

“ Proceed from abreast of the church A , until it is in
“ a line with the white cliff and battery at B ; then, tak-
“ ing their line as your direction, go on till opposite the
“ red church at E ; from which, alter your course, keep-
“ ing the tree H , and the house D , in one line, until
“ abreast of the flag-staff at C , and of the tree at F ;
“ whence stand to C , and when a cable's length from
“ the shore, in six fathoms, take a new direction towards
“ D ; when you are in four fathoms, take your course
“ direct for the house at G ; when coming between the

“ house at D, and the tree at F, go down mid-channel, &c.”

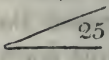
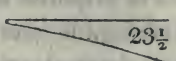
I have not dressed this in sea-language, because it would have required much explanation; I trust it is perfectly intelligible, and that the student will see how much particular determinate points, and lines of direction, (all ultimately deducible from trigonometry), are in use on occasions where its influence is not immediately obvious to persons who only observe superficially.

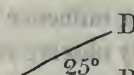
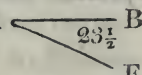
EXAMPLE XXII.

How Surveys are to be taken with the Standard Triangle.

To survey the field C D F E, proceed to nearly its center; or, if any thing, more towards its shorter side, as the angles drawn from the base-line, A B, towards C and D, will be separate, and more on an equality with those towards E and F. Here it is proper to remark, that very acute angles should generally be avoided; open, or obtuse angles, (not too much so), are easier ascertained; for, it must be obvious, that two lines which proceed very close to each other, for any distance, do not afford so distinct a point of contact as two that cross at right angles; in which the exact intersection cannot easily be mistaken.

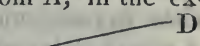
Measure your base-line with great exactness, carefully examining, with your ten-foot pole, as to the correctness of this particular. A long base is a very great advantage, as it makes the angle less acute. Say that the length of your line, (viz. one hundred yards), be your base; leave the line on the ground, properly extended between A and B; lay down your standard triangle at A, with the spike in an exact line towards B, and take the direction of the

line AD, by means of your whip-cord, which being raised, and made to cut the point D, will also cut, on the hypotenuse, the angle BAD  25; reverse the instrument, and take the angle BAF  $23\frac{1}{2}$ in the same way. Note down the angle thus made, in the following manner:

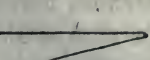
☉ A, to ☉ B, 100 yards: A  25° B; A  $23\frac{1}{2}$ B
F

Observe, the circle with the dot denotes a station; that the angle is always to be drawn in the same direction, as it actually exists, and, that the line from which it is derived, as well as the line made, is always to be lettered as above, whence no mistake can possibly arise. The value of the angle is always to be written in distinct cyphers, between the two lines. The following exemplifications will shew, at one view, how the angles are to be described.

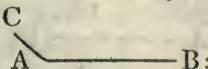
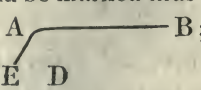
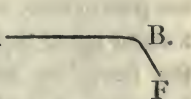
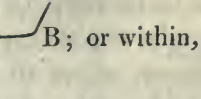
If an angle turns back on the line whence it was derived, it must be acute, and must be described as such in the noting down; an acute angle, from A, in the ex-

ample, must turn to the right, thus; A  B; and, an acute angle from B, must turn to the left,

thus; A  B; but, as they may be measured from either side, as well as from either end, we should be careful to imitate their actual tendency in a suitable manner, by describing such as turn within, from A, thus; A  B; and if from B,

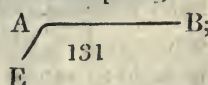
thus; A  B. By this very easy device, we have a regular plan for the noting down of angles, in

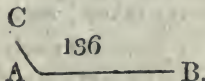
what is called the field-book, without the possibility of mistake.

Obtuse angles would be marked thus; ; or, if within, thus; ; from B, in this manner; ; or within, .

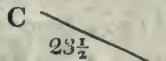
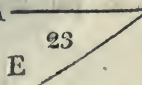
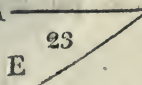
To denote acute angles, 45 degrees will be found a good opening, affording room for the cyphers; and for obtuse angles, about 135° should be adopted.

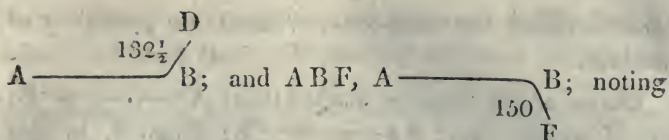
There is usually a point of direction towards the north in most plans, either marked by a flower-de-luce, a long cross, a star, an arrow, or some such thing, indicating the northern point of the compass by its direction. (See Fig. 1 and 2, in Plate 15.) But arrows are, more generally, used to shew the course of currents, their points being in the direction to which the waters run.

But, to proceed; let the standard be again reversed, so that its spike be *from* B, but in a perfect line with the base; the stud, as before, being at the exact point, A. In this position take the angle BAE,  131

then turn the instrument over on its back, and bringing all points to their proper places, take the angle BAC,  136

In this manner all the necessary lines are drawn from the point A, of the base. Carry the instrument to B, and thence, in the same manner, measure off the angles

 $23\frac{1}{2}$ ABC,  23 ABE, A  B; ABD,



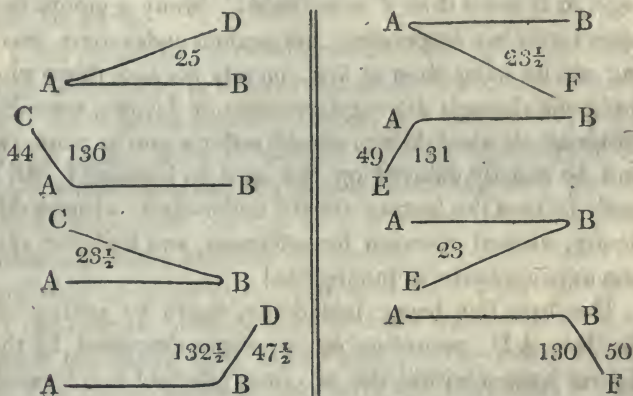
them down, as before, and always keeping the base, or the parent line, as a horizontal; the line last made being thrown off in the proper direction, as may be observed in the foregoing instances, where all the lines are set off from that end of the base whence they are actually seen on the standard triangle; and they are made obtuse or acute, according as they are under or above ninety degrees, besides being set off above or below, so as to correspond with the actual survey.

No measurements are made in this mode of surveying, except with regard to the base, of which the length cannot be too accurately taken; for, if the reality does not, in every particular, correspond with the imitation on paper, not only will the representation of the base be false, but every angle, drawn from its ends, will partake of the error.

We have thus the measurement of a field in a very small compass; in fact, contained in a few figures, indicating angles, which may, at any convenient moment, be brought to appear exactly similar to the grounds in question, as the following process will prove:

Choose a scale of such a size as may suit the paper on which you are about to delineate the survey taken; and, if the extent be great, allow any number of yards for each measure on your scale, observing, that they should either be applicable to ten, or twelve, as multiples, for the more easy appropriation of the scale; thus assume the scale of forty to a foot, on the reverse of your protractor, and say that each of the forty is ten yards. As your base was one hundred yards, you must, after drawing a fine line with

a well-pointed pencil, take ten measures between the points of your compasses, and set them off on the line; this will represent your base, the terminations of which you are to mark slightly, A and B, as they are the points from which you have to draw all the angles you noted down, and which are as follow :



The four in the left column are all angles *from* the observer's base.

The four in the right column are all angles *within* the observer's base.

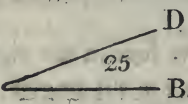
N. B. It is proper to mark obtuse angles within, provided you note the obtuse angle; but, if you only note the complement of 180, that is, the quantity required to complete the angle to the horizontal, you should then mark such complement externally, as above.

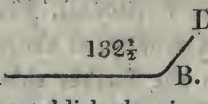
All the angles taken on the further side of the base, are termed *from*; and all on this side, to take which, you must turn round to, in opposition from the former, are termed *within*.

I am aware, that, by thus simplifying the terms, and the manner of noting, as well as of taking the sights, I

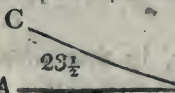
shall attract many an interested or pedantic criticism; but such will not deter me from pulling down the partition that has so long separated ignorance from knowledge! The student may find abundance of treatises, replete with high-flown epithets and pompous arrangements; if *he* can understand, and turn them to advantage, it is more than *I* ever could! Many a month has passed over my desponding, but zealous endeavours, leaving me no wiser than at first, merely because those who had gone through the regular course of fustian, were determined all who followed should suffer a similar penance, and be equally delayed on the road to learning! All I study is, that the learner should understand, without difficulty, without occasion for reference, and find the system as progressive as intelligible!

The base-line being laid down, begin by setting off the line *AD*, according to the angle described by the figures written within the two lines whereof it is formed,

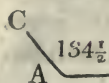
viz.  *AB*. The length is not at this moment of any importance; but it is best to run it out to the extent of your protractor. Then set off the line *BD*, from the other end of the base, according to your memorandum of the angle at which it stood from the extremity, *B*, and which is noted down without the angle;

this will be,  *AB*. This will intersect the line *AD*, and establish the situation of *D*.

Now measure off the line *BC*, forming the angle, as

 *AB*; and, returning

to A, set off the line C A B, as noted down, namely,

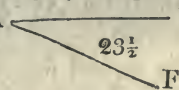


B. These two last-made lines will, by their intersection, give the proper position of C.

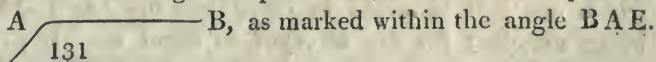
By the foregoing operations, all the lines *from* the base are disposed of, and the two corners on that side of the field are fixed.

I was particular in shewing the formation of each triangle separately, to convey the most immediate idea of their effect to the student; but, for the sake of dispatch, where we are in the habit of working correctly, it is usual, when the protractor is placed at any point, whence two or more lines diverge, to draw them all before that point is quitted; and this is, indeed, by far the quickest and least laborious way; but I should rather recommend to the learner, to finish each triangle separately, than to throw out many lines which may possibly perplex him. As that mode has been taught in the foregoing, I shall take the advantage of the work remaining, to be drawn *within* the base, to explain the other practice.

From A, draw the line F, making the angle equal to what is noted for B A F, viz. A

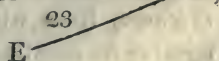


without moving the protractor, draw A E, equal to

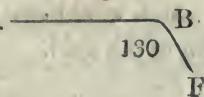


E

Move the protractor to B, and draw B E equal to the noted measure of the angle A B E, A



draw also B F, equal to the noted measure of the angle A B F, A



Here will be seen two intersections, resulting from the lines thus drawn; for the lines AF and BF will cross, and determine the situation of F , while AE will be intersected by BE , at the point where E should stand.

It only remains now to draw the four lines, CD , DF , FE , and EC , which will connect the four corners, CDF , and E , and the delineation is complete.

EXAMPLE XXII.

POLYGONS, of every description, may be surveyed in the same manner, as may be seen by the figures, I have, in each, drawn the proper position of the base-line, AB , and described the proper triangles; but I recommend to the student, that he should practise a variety of such operations, from figures of his own devising; in this manner he will speedily acquire a ready intimacy with the subject, and learn more than could be contained in many ample volumes. He is now arrived at that stage, when he should endeavour to divest himself of every aid, and quit the leading-strings of instruction.

In the second figure, the base-line is carried up to the boundary, leaving room to take the sights from A , for which one foot is added in computing the base; that being sufficient for the surveyor to stand on. If the standard cannot be laid down to the right of A , in the line BA , on account of the angle BAC being acute, it must be drawn back fifty yards into D , the middle of the base, from which a sight may be taken to C , to determine its situation, by DC intersecting BC .

The foregoing mode is given, as the easiest to avoid the necessity of a second station, which would else be indispensable, on account of the form of the field.

EXAMPLE XXIII.

It often answers, and, indeed, is sometimes necessary to lay the base-line even between two corners, or conspicuous objects, in the boundary of a field, as in the following figure.

By drawing the base, A B, between the two re-entering angles, which is easily done, by a person standing at C or D, directing those who hold the cord into their proper line, those points, D and C, are ascertained merely by measuring their distance from the ends of the base respectively; then, as the field is, properly speaking, composed of two areas, it becomes very easy to survey each separately from the same base. This will often be found a great convenience; more especially where the fields are spacious, or where they are diversified by undulations of ascent and descent, or what are commonly called "tops" and bottoms."

That base is generally best, which is not less than one-fourth of the longest sight to be taken; less will do; but then the angles are, in some instances, too acute, and, as before remarked, cannot be so clearly defined, as when the lines cross at right angles: the nearer they are to being mutual perpendiculars, the more accurately will the point of intersection be found. It is better to make a third station, by means of a triangle, as in the next example, than to cramp angles too much, especially if the sights are directed to very remote objects.

EXAMPLE XXIV.

SOMETIMES a variety of stations are needful, not only on account of the unevenness of surface, or the intervention of coppices, huts, &c. but from the irregular

form of the land to be surveyed: the annexed figure may serve as an instance.

Here we have a field so formed, that no situations could be found for *two* stations, competent to take accurate sights to each angle from each end of a base-line; we, therefore, select *three* spots, from which sights can be taken to every corner. To do this, let three flags be placed at A, B, and C; ascertain their different bearings, by measuring the shortest distance, as BC, from them taking sights to A, whereby the measure of the triangle will be known; this being carefully done, take sights to each corner of the field, *from any two stations* that may give the most open angle; that is, the nearest to a right angle.

N. B. A base-line is not requisite in this, as the several distances between the points of the triangle, ABC, are very easily ascertained by sights, in the common way; each side becoming the base-line alternately.

The distance between the stations will be the first thing noted; that is to say, the angles they make with each other. When delineating the survey, they will be the first points for consideration, it being evident, that the whole of the surrounding angles must be laid down on a thoroughly correct basis. If the triangle, ABC, which denotes the three stations, are faithfully committed to paper, he must indeed be a bungler who can mistake the residue of the operation!

EXAMPLE XXV.

WE are not always to expect level plains, or that our work is to be computed as level. Where rising grounds occur, they must be measured, not only as to distance, but in respect to their heights, which are to be allowed for in the delineation on paper, otherwise the different parts will not combine.

To explain this more clearly, let it be considered, that the hypotenuse is always longer than either the base or altitude of a triangle; and, that the ascent, or sloping side, of every hill, is the hypotenuse of a triangle, of which a horizontal line, drawn from its foot to the spot under its utmost height, is the base; and a perpendicular drawn from that utmost height, meeting the end of the base at right angles, (as must ever be the case), is the altitude.

Let ABC be the ascent, and descent, of a hill, whose summit, B, is forty feet above the level, AC; from the summit, B, draw the vertical perpendicular, BD, meeting the level line, AC, at right angles, in D.

Now, the lines AB and BC brought down to the horizontal, or level line, AC, will overlap, and cannot lay within the length of the line AC; for, being each an hypotenuse, their joint lengths must exceed their joint bases, by as much as overlaps between the dotted lines E and F.

This proves the necessity of ascertaining the height of every hill, as well as the measurement of its several ascents, or sides; because we can, by this means only, correct the error which would otherwise obtain, and totally derange every part of the calculation.

EXAMPLE XXVI.

As a gradual introduction to the manner of using the quadrant, or scale, on the hypotenuse, in taking heights, &c. I will instruct the student how to take the breadth of a river, at one sight.

Place the instrument, as nearly as may be practicable, over the water's edge, at B, the angle projecting from you; measure the perpendicular height of the bank you stand on, from the level of the water.

The standard being exactly perpendicular, find the

angle between the horizontal limb, and the place cut on the hypotenuse, by the whip-cord, when seen in a perfect right line from the stud, directed to the edge of the water, at the foot of the opposite bank, A.

Say the height of the bank be twenty-four feet, and the angle made be ten degrees and a half; add the height of your instrument, viz. five feet, three inches, to the height of the bank, (*i. e.* twenty-four), making in all twenty-nine feet, three inches; erect the perpendicular, C D, Fig. 2, on the line D E, which represents the level of the water; above which, the point C stands twenty-nine feet, three inches; then, from C, draw the descending line C F, at ten degrees from the horizontal, and the point F will determine the breadth of the river, viz. one hundred and fifty-five feet.

But this could not be done to a certainty, and without recourse to further operation, unless the line D E were a perfect level, and the height of the bank C D known, without recourse to other operations.

EXAMPLE XXVII.

HEIGHTS, whose distances are known, may be measured in the same manner; or where the height is known, the distance may be discovered.

This is the reverse of the foregoing example, for we now have to take an ascending angle; therefore the projection of the triangle must be *towards* the surveyor, who takes his sight *from* the hypotenuse, so as to bring the object, B, the stud on the instrument, and the angle demonstrated on the quadrant, all in one line.

Say, the distance from A, to the flag-staff at C, is one hundred yards, and that the line drawn by the whip-cord, (extended from the stud to the hypotenuse), indicates an angle of seven degrees. We draw a line, diverging

seven degrees from the horizontal, and erecting a perpendicular, CB, at one hundred yards from A, the intersection of them, *i. e.* of AB and CB, will give the height of the flag, twenty-eight feet.

By the same means, if we know the staff, BC, to be twenty-eight feet high, we should find the intermediate distance; for, by taking the angle of its altitude, and applying its known height as a perpendicular to a base-line, thereon moving it until it should touch the line of sight, we should ultimately discover its remoteness from the foot of the standard triangle.

This operation is also performed at one sight, but it can only be so done when one of the sides is given.

EXAMPLE XXVIII.

IN general, two sights are taken, where either altitude or situation, is to be ascertained; in such cases, the base-line is made to run in the direction of the sights, one station being nearer to the object than the other.

AB is the base-line, say one hundred yards in length; from each end of which a sight is taken with the standard triangle; the hypotenuse towards the surveyor; both sights will give ascending angles; but that next the church will be the least acute from the horizontal, ABC.

This mode will not only give the height of the steeple, but will ascertain the precise spot over which its center is upheld, and which may be known by the perpendicular, DC.

This is of peculiar use among military men and others, it being often of the utmost importance to know the heights of batteries, &c. whereby the angle to be made in the elevation of cannon, &c. may be regulated.

Although horizontal sights are, assuredly, of immense

service in every branch of surveying, and, that, in reconnoitering, they prove of very great advantage, yet we have not always the means of taking such; in which case, the summits of conspicuous parts of fortifications being taken, may lead to an understanding of the general plan. As this mode of survey may be carried on principally behind a rampart, it is, for such purposes, more generally useful than horizontal sights only, for they give nothing more than distance, while that above described gives altitude also.

EXAMPLE XXIX.

THE foregoing example may be beautifully and usefully illustrated herein.

It is required to know the heights of the two batteries, A and B, from the stations C and D, separated from the fort by an arm of the sea.

From C, take one sight to A, and another to B; do the same from D; this will give their perpendicular heights, which being delineated thus, will also shew their horizontal distances from C and D, with their horizontal, and their inclined distances from each other. Fig. 2 exhibits their distances and altitudes.

EXAMPLE XXX.

IN the following figure we will suppose the land to be almost a square pyramid, or conical, as such will come easiest to illustration. The point A, in the center, being higher than the boundaries BCDE, we must ascertain the exact altitude for which allowance is to be made; otherwise we should fall into the error described in Example XXV.

Set up a pole at A, and put the target at such a height

thereon, as may be visible from each of the four corners, B, C, D, E; if five feet, three inches will do, that is preferable, as it corresponds with the height of the standard triangle, and saves some calculation.

Fixing the instrument perpendicular, at either corner, say E, with the hypotenuse towards you, and the standard towards A, take your whip-cord, and ascertain the angle made by the ascent, observing what degree is cut thereby on the hypotenuse, when the cord appears in a right line with the stud, and with the white line on the target.

Fig. 2 will shew my meaning completely.

This will determine the angle made by the ascent, provided the target and the standard be, as recommended, equidistant from the ground; otherwise the surplus on the target-pole must be deducted.

Another sight, taken from the same spot, towards D, will ascertain, by means of a ten-foot pole, or a wand and flag, what may be the difference of height between D and E, by the same operation as is detailed in Example XV. and others; if any, it must be noted; say D be one yard lower than E; measure the angle AED, laying the standard down, with the spike towards D; likewise measure the distance between E and D; remove the standard triangle to D; from which, take another sight to the target at A, in the same manner as was done at E. This angle will vary a little from the former, being more obtuse from the horizon, on account of D being three feet lower than E. Now measure the angle ADE, laying the standard down with the point towards E.

By this process, you have the altitude of the summit A, above the levels of E and D; the other corners are to be treated in the same manner, if particularly required, or that there is much difference in the level of the boundaries, otherwise, by measuring their several sides only,

the area of the field may be known. Example XXV. shews, that the lines of ascent are greater than their horizontals, consequently, when laying down the survey on paper, the horizontal space is to be the guide, and the extent of the hill is to be marked by lines of acclivity, as in Fig. 3. It may be useful to note the altitude above the ordinary level of the adjacent lands, by figures on the summit, as above, which implies an elevation of 44 feet above the boundary. If a hill is long and winding, its course should be denoted by lines of acclivity proceeding with it, on each side of its direction, as in Fig. 4.

Water is commonly distinguished by a number of horizontal lines along its borders, as in Figs. 5 and 6.

OF EXTENSIVE SURVEYS.

It is now time to instruct the student how to conjoin the surveys of various fields, or compartments of land. To do this with facility, and to avoid confusion, are important matters; therefore the utmost care should be taken, in the outset, to establish particular points, whence all that follow may appear in a regular continuation, free from intermixture, and in many respects deducible from the origin, in an open and unerring track.

The first field surveyed should have, either in it, or on its boundaries, something remarkable, visible from the next, or other fields, and should serve as a rallying-point for all the diverging operations. If each field be not already designated, some term should be applied, for it is of much service to the surveyor to have an aid of that description, intelligible to the proprietor, and, of course, contributing to his understanding of the plan. If there be any particular building, or other object, by which the same intelligence can be conveyed, it should, for want of prenomination, be adopted; it is far preferable to field

N^o 1, 2, 3, &c. which does not give that ready idea of particular localities implied by figures, or by names, more respectively applicable.

EXAMPLE XXXI.

LET it be required to survey the estate contained under the annexed figure.

First, survey Brent's Meadow, (the hay-stacks being an object), and lay it down in a small sketch, as in Fig. 2. Proceed to South Fold, which lay down next thereto, taking care to ascertain where the corners of the hedges to the north and south of Brent's Meadow, fall in on the east line of South Fold; proceed to Rot Nook, and ascertain how it joins the South Fold, especially at the corner next Brent's Meadow; then take the contents of Norton's Close, observing where its boundary, towards Norton's Ridge, is cut by Brent's south boundary. In taking Norton's Ridge, you must be accurate, and mark carefully where the south line of Brent's Farm meets it on the east. Take Brent's Farm next; and, as its north boundary applies to Old Close, proceed to Old Close, and take the whole north line, up to the west corner of North Fold, setting off the ends of the partitions between that and Harper's Bottom, and Nine Acre, respectively.

Returning from North Fold, enter Harper's Bottom, which being surveyed, will give the west side of Nine Acre, and enable you to finish therewith.

The reason for doing this is, that by getting one side of a field which is common to another, one side of the inclosure next to be surveyed, is obtained, and its connection with the preceding, established. Thus, having taken Brent's Meadow, we find no one of its sides equal to the side of the next field, we there-

fore enter South Fold, as being at hand; and, having a pond in the side next to the hay-stacks in Brent's Meadow, it furnishes us with the exact spot where the north boundary of Brent's Meadow terminates: by this, the exact relative situation of South Fold is established. The boundary to its south gives one side of Rot Nook, which is thus equally well certified. In entering Norton's Close, we observe that the east end of its northerly hedge agrees with the corner of Brent's Meadow, which establishes the point for its corner in that direction, *i. e.* the north east. Again, in Norton's Ridge, the hedge running north and south, ends in the south, even with the termination of Norton's Close, consequently that point is also established. The short side, where the pond is, towards Brent's Farm, being common both to that and to Norton's Ridge, establishes the situation of that side; and, as the north side of Brent's Farm measures one side of Old Close, the latter is easily united to that farm.

As the whole of the north is in an even line, we take one long view from the end of Old Close to the west of North Fold, of which we ascertain the length of the north side, as also the length of the north side of Harper's Bottom; having this, the North Fold and Harper's Bottom are easily put into their proper places. It is needless to survey the Nine-acre Field; for, by the survey of those around, it has become insulated, the boundaries of the adjacent fields being the boundaries of the Nine Acres.

While noting down the contents of each field, it is proper to make a memorandum as to which of its sides is to guide, for the adjunction of the next field: if a side is to be the director, let it be marked with a cross, or a dash, through it; and, if a corner is to guide, let it be distinguished by a small angular inflexion. Of these, specimens are given in Fig. 2, where each side, or cor-

ner, that is to give the line, or point of connection with the next, is thus characterised.

The memoranda would stand thus at the head of each field:

FIRST STATION.—BRENT'S MEADOW.

Two hay-stacks, N. W.

Medium length, 192 yards, } nearly a parallelogram.
Medium breadth, 138 D^o }

A pond in N. W. corner, common to S. F. and H. B.

	192				
	138				
	1536				
	576				
	192				
	4840	)	26496	(	5 1 35 27 $\frac{1}{4}$
			24200		
	1210	)	2296	(	1
			1210		
			1086		
			4		
	121	)	4344	(	35
			363		
			714		
			605		
	4	)	109		
			27 $\frac{1}{4}$		

N. B. The calculations are added to each station, for the sake of perspicuity, though they would of course be made when the whole survey were taken.

SECOND STATION.—SOUTH FOLD.

Adjoins S. E. corner with S. W. corner of Brent's Meadow.

South side, common to Rot Nook.

North side, common to North Fold.

Diagonal 225 Yds.

N. W. offset 76 Yds.

S. E. D° 100

2) 176

88

225

88

1800

1800

4840) 19800 (4 0 14 16½

19360

1210) 440 (0

4

121) 1760 (14

121

550

484

4) 66

16½

N.B. Offsets from diagonals are marked thus:  : the longest line denotes the diagonal, and the short stem gives its deviation from that diagonal, according to its real direction. The figures attached to the offset, thus,  shew the number of yards from the outset of the diagonal, where the offset strikes off, and are placed accordingly, to the right or left of the stem. In this manner we should estimate the contents of

Fig. 3,  denotes that the offset, C, is taken at 47

yards from the point A, in the figure; and  would imply that the offset, D, was 53 yards from the point B.

THIRD STATION.—ROT-NOOK.

North boundary, in common with South Fold.

	Diag. of trapezium	170	Trap.	170
				65
	N. W. offset	65		850
				1020
	S. E. D ^o	65		11050
		65		
				136
				29
	South triangle.			1224
	Base	136		272
	$\frac{1}{2}$ alt.	29		3944

South offsets:

							42
						32	
						28	
	40	Base, 110					
		Alt. of one at 40	32	40		256	42
	68	D° of one at 68	32	16		64	16
					640		
						896	672

	Sq. Yds.
Trap.	11050
Triangle	3944
	640
The 2 offsets produce 3 spaces, viz.	896
	672

		A.	R.	P.	Sq.Yds.
Total area	4840	)	17200	(	3 2 8 13
			14520		

1210	)	2680	(	2
		2420		

)	260
	4

121	)	1040	(	8
		968		

4	)	52
---	---	----

13

FOURTH STATION.—NORTON'S CLOSE.

S. E. corner of Brent's Meadow joins N. E. of this.—North side, common to Brent's Meadow.—Ranges on South side with Norton's Ridge.

N. W. diagonal . . . 236

S. W. offset 86

N. E. D° 90

2) 176

88

236

88

1888

1888

A. R. P. Sq. Yds.

4840) 20768 (4 1 6 16 $\frac{1}{2}$

19360

1410) 1408 (1

1210

198

4

121) 792 (6

726

4) 66

16 $\frac{1}{2}$

FIFTH STATION.—NORTON'S RIDGE.

S. W. corner joins S. E. of Norton's Close:—N. E. triangle makes S. W. side of Brent's Farm.—Western hedge common to that, and to Brent's Meadow.

	N. W. diagonal . . .	280	280
			92
65	Offset, N. E. . . .	86	560
			2520
248	D° S. W.	98	25760
		2) 184	
		92	

	Triangular slip, whole length of West side . .	255
187	$\frac{1}{2}$ altitude	8
		2040

	Triangular slip, S. E. corner	130
145	$\frac{1}{2}$ the altitude	19
		2470

Triangular slip, N. E. corner 82

15 $\frac{1}{2}$ the altitude 6 $\frac{1}{2}$

1 offset from S. E. slip 54 41

$\frac{1}{2}$ altitude 5 $\frac{1}{2}$ 492

1 Do. 74 27
 $\frac{1}{2}$ alt. 3 $\frac{1}{2}$ 220

37 447

222

259

Trapezium 25760

West slip 2040

S. E. slip 2470

N. E. Do. 533

1 offset 447

1 Do. 259

A. R. P. Sq. Yds.

4840) 31509 (6 1 8 17

29040

1210) 1469 (1 $\frac{1}{2}$

1210

259

4

121) 1036 (8

968

4) 68

17

28

SIXTH STATION.—BRENT'S FARM.

Short side on East of Norton's ridge, common to this on S. W.—

North side makes South of Old Close.

South triangle, base . . . 180

 $\frac{1}{2}$ altitude . . . 31

180

540

5580

East triangle . . . 198

 $\frac{1}{2}$ altitude . . . 62

396

1198

12376

West triangle . . . 198

 $\frac{1}{2}$ altitude . . . 22

396

396

4356

East slip . . . 138

Mean of offset . . . 7

866

South triangle . . .	5580
East Do.	12370
West Do.	4356
East slip	866
Total	23172

		A.	R.	P.	Sq.Yds.
4840)	23172	(	4	3	2' 21 $\frac{1}{2}$
	19360				
1210)	3712	(	3		
	3630				
	82				
	4				
121)	328	(	2		
	242				
4)	86				
	21 $\frac{1}{2}$				

SEVENTH STATION.—OLD CLOSE.

North side of Brent's Farm common to this on the South.—

North side of Old Close in a line with North sides of Harper's Bottom and North Fold.

South triangle . . .	122
122 $\frac{1}{2}$ altitude . . .	21
	2542

East triangle 153

$$\begin{array}{r}
 82 \overline{) 153} \\
 \underline{164} \\
 76\frac{1}{2} \\
 \underline{918} \\
 612 \\
 \underline{7114\frac{1}{2}}
 \end{array}$$

North triangle 224

$$\begin{array}{r}
 86 \overline{) 224} \\
 \underline{172} \\
 52 \\
 \underline{416} \\
 672 \\
 \underline{8848}
 \end{array}$$

East slip 138

$$\begin{array}{r}
 55 \overline{) 138} \\
 \underline{110} \\
 28 \\
 \underline{165} \\
 966
 \end{array}$$

South triangle 2542

East Do. 7114 $\frac{1}{2}$

North Do. 8848

East slip 966

A. R. P. Sq. Y

$$\begin{array}{r}
 4840 \overline{) 19500\frac{1}{2}} \\
 \underline{19360}
 \end{array}$$

$$\begin{array}{r}
 1210 \overline{) 140} \\
 \underline{4}
 \end{array}$$

$$\begin{array}{r}
 121 \overline{) 560} \\
 \underline{484}
 \end{array}$$

$$\begin{array}{r}
 4 \overline{) 96}
 \end{array}$$

$$\begin{array}{r}
 24
 \end{array}$$

EIGHTH STATION.—NORTH FOLD.

South side common to South Fold, on the West, in a line with North Fold and Rot Nook.

N. E. diagonal . . . 218

195 N. W. offset . . . 95

1106 S. E. Do. 79

2) 174

67

218

67

1426

1308

A. R. P. Sq.Yds.

4840) 14506 (2 3 39 16 $\frac{1}{4}$
9680

1210) 4826 (3
3630

1196

4

121) 4784 (39
363

1154

1089

4) 65

16 $\frac{1}{4}$

NINTH STATION.—HARPER'S BOTTOM.

North side lies even between North Fold and Old Close.—South side common to Brent's Farm.

N. E. diagonal . . . 309

$\frac{94}{1}$ N. W. offset . . . 133

$\frac{106}{1}$ S. E. Do. . . . 64

2) 197

98½

309

98½

154½

2781

A. R. P. Sq. Yds.

4840) 27964½ (5 3 4 13

24200

1210) 3764 (3

3630

134

4

121) 536 (4

484

4) 52

13

TENTH STATION.—NINE ACRES.

Bounded by Harper's Bottom on the West, of which the East side is common to this: other sides all irregularly bounded.

N. E. diagonal	274				
193 N. W. offset	114				
1128 S. E. Do.	114				
	2) 228				
	114				
	274				
	114				
	1096				
	274				
	274				
	4840) 31236	(6	0	39	16½
					29040
	1210) 1196	(0			
					4
	121) 4784	(39			
					363
					1154
					1089
	4) 65				
					16½

We now proceed to sum up the entire contents of the nine fields, thus:

	Acres.	Roods.	Poles.	Sq. Yds.
Brent's Meadow	5	1	35	27 $\frac{1}{4}$
South Fold	4	0	14	16 $\frac{1}{2}$
Rot Nook	3	2	8	13
Norton's Close	4	1	6	16 $\frac{1}{2}$
Norton's Ridge	6	1	8	17
Brent's Farm	4	3	2	21 $\frac{1}{2}$
Old Close	4	0	4	24
North Fold	2	3	39	16 $\frac{1}{4}$
Harper's Bottom	5	3	4	13
Nine Acres	6	0	39	16 $\frac{1}{2}$
Total	47	2	5	0

The student should remark, that where his compass, or, for want of one, his observation, may fix objects in any of the cardinal points, as East, West, North, or South, they should be so designated; and that all diagonals are to be classed either as *North East* or *North West*; all between N. and W. being N.W. and all between N. and E. being N.E. The same in regard to points, between South and West, and South and East, respectively. He will find this prevent much embarrassment and confusion.

In surveying on a larger scale, attention must be paid to the more principal features within the compass of observation, such as obelisks, summer-houses, cottages, large trees, rocks, banks, mounds, park and garden walls, or other objects generally conspicuous.

The survey of these proceeds exactly the same as in the case of a large field, by division, into trapezia and triangles; the outline being accurately obtained, the interior is afterwards arranged according to the several sub-

divisions of which it is composed. The windings of streams, the track of roads, the bridges, &c. must all be carefully noticed, and placed in their exact situations, with every indication necessary for the information of the proprietor, and of all who may have an interest in the survey: nor should the adjoining estates be unnoticed.

One point should ever be held in mind, namely, that at least the majority of surveys are taken under the intention of sale. This should cause the surveyor to be particularly attentive to the general occupancy, the nature of the soil, &c. in the several parts of the estate. *Lime-stone, coal, marl, sand, and gravel-pits*, should be mentioned; *meadow-land* should be so described; and *arable portions* ought to be distinguished by an imitation of furrows; *wood-lands* ought to be known by drawings of *trees*; *hedges*, by *shrubbery*; and for these last it is usual to allow four feet in diameter for their whole length. *Walls* are distinguished by two parallel lines, one being blacker and thicker than the other, as a relief to the appearance, and to imitate shading, thus: . *Poles* are commonly marked by dots, thus: and *rails*, with *posts*, by black lines, divided by larger dots, thus: .

I shall speak more fully of the manner of laying down, and taking the plans of *houses*, when I treat of the survey of smaller areas, such as farm-yards, &c. This branch applies more generally to the architect; but every surveyor ought to be acquainted with a subject so intimately blended with his own profession. Previous to embracing that topic, I shall make some remarks on several instruments, either now in use, or that have been employed in surveying. It cannot be expected that I should describe, or even enumerate all that have been invented for that purpose; indeed, there must have been numbers which, though answering for the amusement of

the inventors, could not come within the circle of general utility.

OF GUNTER'S CHAIN.

THOUGH my objections to this are insuperable, yet, as many students may be of a contrary opinion, and as, at all events, it is proper every learner should know its construction, I shall give it as concisely as possible.

It is 22 yards in length, and consists of 100 links, each 7, 92 inches; it is divided into 10 portions, of 10 links each, and each portion is distinguished by a corresponding number of points, filed on the edge of a small brass plate, affixed to every tenth link. These points indicate the number of portions; the first plate, from each end, having one point; the second plate, from each end, has two points; the third has three points; the fourth has four points. The center plate is larger than any of the others, and is, besides, round and plain; it stands for 50. Thus, if it were required to count 27 links, you may begin at either end; set off two divisions, each of 10, by taking the plate with two points, and thence counting on seven links more. If you have to count 72, either deduct 28 from the other end, or, counting the middle round plate as 50, take two more divisions beyond it, which will be 70; and two links more will complete the measure. This, indeed, is so simple, that it cannot be too much approved. My objection arises from the fraction in every link, which, though not the least difficulty to a proficient, certainly presents an impediment to the novice, and makes the work rather prolix. A chain is, confessedly, better than a rope, because it keeps to an uniform length; but, instead of 66 feet, I should prefer a chain of 100 feet.

As to the calculation of 10 chains making a furlong, and 80 chains making a mile, that is foreign to the purpose; what we generally want of a limited, or portable measure, is the application to *small* distances. The division of yards into feet, and feet into inches, every clown knows; but, when Gunter's chain is used, the rustic assistant must serve an apprenticeship before he can readily understand the measurements he is taking.

Gunter's scale is, assuredly, a compendious instrument, and, so far, cannot be over-rated; but it is a great pity so much pains were taken to form calculations founded on the decimal system above alluded to, which was, in fact, adding another barrier to oppose against unlearned ambition. The inventor would, perhaps, have derived considerable satisfaction, (though I felt chagrin and regret), could he have heard a young man, of no despicable understanding, and some education, say, in answer to my question, *Whether he understood surveying?* "Oh, no, Sir, that is more than ever I shall be master of; for I do not understand Gunter's scale, and Mr. C—— says I never can do without it."

OF THE PLAIN TABLE.

THIS usually stands on a tripod, like a theodolite, and consists of a board about 20 inches by 15; on the outside of which, a graduated frame slips on, much the same as the drawing-frames that fit around a board, to keep the paper fixed and level: in the plain table it is for the same purpose; and, having the gradations marked around the edge, the angles made from the center may be set off. For this purpose, there is a brass scale of about two feet long, and two inches broad, with an upright at each end, about four inches long. In each of these uprights

is a small sight-hole, and above it a slit, in the center of which a horse-hair, or a piece of sewing silk, is fixed, at a due degree of tension. The surveyor, looking through the hole, (at either end, as may be requisite), directs the thread at the other end of the scale, so as to cut any object to be observed. To do this with facility, he places the point of some sharp instrument through the paper, into a brass plate in the center of the board; in the plate there is a small mark, like the studs in a sector, &c. to receive the point, which is exactly in the center; this serves to guide the scale, so that it should not pass the center while taking the sight, which else could not be easily avoided, without much delay and trouble.

There is also a compass attached by a slide to the side of the plain table, having a brass rim, graduated with 360 degrees, both backwards and forwards, so that the surveyor may count either way; and, as the board moves by a ball and socket, fixed under its center, to incline towards any object below its level, and that there is also a fixture for a plumb-line, this instrument certainly is extremely useful for surveying on a small scale, especially within areas, where the angles are numerous, as may be seen in the annexed plate. The brass rule, above described, has generally a line of chords, and various sized scales engraved thereon; their utility would be greater, were it not that, from being very subject to corrosion, by verdigrease, the lines and divisions are soon impaired, and lead to great errors. This instrument is usually sold with a Gunter's chain.

EXAMPLE XXXII.

To survey with this instrument, the principal angles of a field, &c. are observed by means of the sights on the

scale; the assistants measuring, from the plumb under the tripod, to each angle, or point of observation, with the chain; thus, if the plain table were fixed in the middle of an enclosure, as in the annexed figure, it is evident that, by following the direction of the angle made on the surface of the table, the direction and distance of each corner, &c. would be decided.

The surveyor, having taken various sights, and the measures to the respective objects being ascertained by the assistants, he would reduce them to paper, as he went on; for the student is to understand, that the paper is fixed to the table by means of the moveable graduated border, and, that as the brass ruler lays in the true direction from the center, each angle is marked down along its edge, as the sight is taken.

Now, in this no calculation is requisite, neither need the value of the angles be recorded; they are laid down, at the moment of survey, on the paper, with a memorandum of the length of each. If a pair of compasses are at hand, the several lengths of the lines drawn may be set off; then, by joining their terminations, the boundaries are described on the spot.

Surveys may be taken to a great extent with the plain table, as any number of stations may be made thus: having finished your work in one field, take a sight towards any convenient spot in another inclosure, either through a gap in a hedge, or by means of a gate, &c. The chain can be pushed through the gap by an assistant with a pole, having a notch in its end, or by tying the chain thereto. The angle and distance of the new station being taken, are carefully laid down, and measured on the paper; and the graduated frame being taken up, the new station is brought to the center of the board, and the frame is put down again.

In doing this, the paper must be put forward in an

exact parallel to its former position, which is easily ascertained by means of the ruler, or scale; for that being every where of an equal breadth, may act as a parallel ruler, under the guidance of a good eye.

EXAMPLE XXXIII.

THE following survey of a farm-yard, will give the student a complete idea of this useful instrument, the operations of which are extremely simple, and, as far as depends on level work, sufficiently accurate; but, when angles of altitude, or of depression, are to be taken, the plain table is of no use. The price is also an object of some moment: I believe, the general cost of a plain table, complete, is from four to seven guineas, according to size, finish, and the *et ceteras* furnished therewith.

In the foregoing, A and B are two stations, from which sights are taken to each projection, or angle; and the figures, or rays of sight, shew their distances, in feet, from each station, respectively. The internal measurements are taken with a rod, and, in general, are afterwards set off separately on a larger scale, especially the plan of the house, which may be originally laid down with the greatest accuracy, from a plan fastened to the plain table.

The use of the plain table is, in this instance, very great; but, it must be remarked, that the whole of what is done thereby, may be effected with, at least, as much exactness by the standard triangle, the manner of using which, on such occasions, is very simple and certain; it is described in Fig. 2.

Let a cross be made on the ground, either by two lines drawn with chalk, or by means of two pieces of thin cord, stretched at right angles. From the center, formed by

their intersection, measure the distances to projecting objects; as in the former instance, and at each measurement apply the standard triangle, laid flat, with the stud exactly over the center of the cross; the chain, or measure-line, will demonstrate the angle on the graduated hypotenuse: The standard triangle may be turned in any of the four directions indicated by the limbs of the cross, but must always lay with the *inside* of the standard along one line, and the inside of the horizontal, or sight-side, laying along another, at right angles thereto. The four lines will each indicate a quadrant, and their several terminations should be marked as above, to guide the surveyor in his memoranda, as to the angles taken from the center of the cross, through the hypotenuse, in whatever direction it may be laid.

The second, or third stations, where such are necessary, are to be set off from the point, by due observation of an angle; the distance between the centers of the stations, regulated by the angle at which it is received by the second, or other station; will give great certainty as to the correct continuation of the work: thus, in the figure annexed, the line of distance leaves the first station at an angle of one hundred and thirty-five degrees, and is received at the second station at an angle of three hundred and fifteen degrees; consequently, both the crosses are, in every instance, parallel; but this is not necessary, though more sightly *on paper*. See Fig. 3.

I presume, nothing can be easier than what is here described; and, when it is considered that, even according to the most expensive way of proceeding, a standard triangle need not cost more than half-a-guinea; that the instrument is, in every respect, capable of taking levels, heights, and distances, to any extent; also, that by making it on a smaller scale, with hinges and pins, so that

the angular part may be laid flat with the standard, as in the figure given herewith, it will not be too much to say, that it merits considerable approbation, and is worthy to be adopted by surveyors in general, or at least by all such persons as may not possess theodolites. See Fig. 4. Plate 19.

C G is the height of the standard; at A is a substantial hinge, connecting the horizontal, or sight-limb, with the standard; B is a tenon, which turns into the mortice C, when the sight-limb is brought up square. At D the end of the sight-limb is rounded, and admits half the thickness of the hypotenuse, which is let into the middle of D, where it is kept in by a nutted pivot, on which it turns. The end, F, is slanted off within, and is shod with iron, so as to fit into the shoulder at E, where it is bolted through with a nut, which, having wings, can be screwed tight at pleasure; the spike-end, G, is the same as usual, and the stud is, as before, fixed by screws, (passing through both its own plate, and the reverse perforated plate), to the top of the standard.

This requires to be made of harder wood than the fixed standard triangle, on account of the tenons, mortices, screws, pivots, &c. all of which weaken the instrument in some degree, and, in rough hands, might subject it to injury. This construction, however, gives it all the advantages of a portable machine, and, though not quite so accurate as a theodolite, many points of preference, among which, the ease wherewith it may be made by *any common artificer*, is not the least.

OF THE CROSS STAFF.

THIS is a very old, and, I thought, an almost obsolete invention, until I met with a treatise on land surveying, by

William Davis, in which, I observe, it is introduced to notice, in a very strong manner, as being the principal instrument used by him for taking surveys; but that work is nothing more than a compilation from various old authors, especially from *John Hammond*, who published, in the year 1725, *The Practical Surveyor*, which *he* acknowledges to be only a compilation! Other parts of Mr. Davis's volume are taken from an anonymous publication, by H. C. *Gent.*; printed by T. Bennett, of St. Paul's Church-yard, 1706; and the mensuration of artificer's work is extracted, almost verbatim, from an octavo, written by *William Hawney*, Philomath, and recommended by the Rev. Dr. John Harris, F.R.S. This little volume was printed by a society of booksellers, in 1729, and was designated "*The complete Measurer, or the whole Art of Surveying.*"

In these several books, which, in their day, were first-rate publications, though in quite a modest style, the cross-staff is mentioned as the common instrument then in use; for the theodolite is there spoken of as a new and wonderful discovery. How such an insignificant thing as the cross-staff could have been the basis of a treatise on surveying, in these times, is a matter of surprise. The instrument is utterly defective, as will be seen from the following description; and the parade made about its utility, is much of a piece with the many successive pages of figures with which Mr. Davis's abounds; printed too, in such a loose manner, as at one glance convinced me there was a plentiful lack of *useful matter, wherewith to swell the volume!*

The truth is, that Mr. Davis's book might have been handsomely printed in one-third of its present bulk, whereby, at least, its convenience, as a compendium, would have been augmented, even if the price had not

been proportionally lowered. Further, that treatise is by no means calculated for a perfect novice; nor are all the operations duly prepared, by previous instruction, as to many of their component parts; but this, I apprehend, may have been an oversight, in the hurry of getting forward with a volume, which may be compared with Peter Pindar's razors, that were made not to *shave*, but to *sell*! We might, however, have expected, in a fourth edition, now extant, to have seen those irregularities corrected!

The cross-staff takes its name from a piece of board about six inches square, and an inch, or more, in thickness, which being affixed to the top of a staff of five feet, six inches long, by means of a strong screw through its center, is level with the horizon, when the staff is perpendicular. This board has two small furrows cut in it, by a sharp instrument, so that they should be clean, and free from raggedness: they are cut diagonally from the opposite corners, in form of St. Andrew's cross. Their use is, to direct the eye to a perfect right angle; but, owing to their being so short, they do this but imperfectly. Studs at each corner are generally preferred.

All that can be done with the cross-staff, is to take a right angle. Need I say more, to convince any reader of the inadequacy of the instrument to even the survey of a farmer's yard? or, must I enter into a detail of all the risings and fallings of land, on an estate of moderate extent; the various obstacles that present themselves, and the want of something to ascertain a proper perpendicular, or level? Indeed, I do not see how it is possible, with a cross-staff only, to trace a diagonal over a trapezium, whose surface is undulated above the height of the instrument! A clod-hopper, or ploughman, would have a far better chance with his line of poles, as fixed for drawing his first furrow!

But this is allowing the instrument to be perfect, which it never can be. Say the grooves be but one-sixth of an inch broad, and as much in depth. In so short a line as eight inches and a quarter, which will be about the diagonal of a square of six inches, the angle may be false, to the extent of nearly *two degrees*, which, in a remote sight, will make a great difference, unless, (as is sometimes done by *eminent* surveyors), the parts are pushed into their places with some little violence, or, perhaps, after trimming them here and there, so that they may *come in*, as it is termed!!!

It looks well to see a volume ornamented with plates, describing instruments, of which the qualities are blazoned forth with great pomp and plausibility; but, I fear, instead of the cross-staff being such as Mr. Davis would lead us to imagine, it will be found that, (like many *infallible* implements of husbandry), in lieu of suiting *all soils*, the soil must be suited to the instrument.

We may admire even absurdities, provided they are novel; for it often happens, that but little is wanting to bring that which has failed to perfection; and we commonly find our best inventions have been derived from hints afforded under such circumstances. But, really, I know not of any excuse for ransacking odd volumes, for the purpose of starting, *as novelties*, what have been tried and *condemned* in former times. It looks not only like a want of genius, but a want of modesty: especially when the authorities are neither quoted nor acknowledged!

OF ROADS.

THIS branch of surveying generally requires a compass, for the purpose of shewing the various turns, which would be tedious to measure with any other instrument. The circumference should be graduated, as in the plain table, by which means small angles may be taken. For general purposes, such as putting down mile-stones, &c. a perambulator is an excellent instrument! There are various kinds; but that which is pushed on before a man, who holds by the handle, and directs the wheel, is the best. It has a dial of metal, with three indices, like the hands of a clock; one shews the miles; the second shews the furlongs, and the third shews the poles gone over; they are generally very correct, moving by clock-work.

This best kind of instrument, in respect to accuracy, is the invention of Mr. Tugwell, of Bath, and consists of a wheel of iron, the circumference of which measures one perch, or sixteen feet and a half, consequently, is five feet and a half in diameter. As it turns, it gives motion to a long axle, on which are as many threads of a screw, as there are poles in a mile, viz. three hundred and twenty. After proceeding a mile, the perambulator is checked, by coming to the end of the screw, when the axle is taken out, and, the contrivance-being reversed, the instrument can be again propelled by the man who manages it, and who takes hold of a bar near the axle, and parallel thereto.

The late Colonel Jacob Camac, however, made a perambulator of the same dimensions, but on a more simple

plan. His had the same number of revolutions, but the axle was very short, and worked in a pipe of iron, which served the conductor as a hold. The screw went through a nut, in which was a female screw, and the nut had a stud that appeared through a slit nearly the whole length of the pipe. The nut was thus prevented from turning round, and was consequently propelled by the motion of the wheel; and, in its progress along the slit, indicated, by figures on its margin, what number of furlongs had been gone over. When the nut arrived at the end of the slit, it indicated one mile, when, merely by turning the wheel round, so that the conductor appeared to walk on its other side, the nut would return to the other end again; and thus, without the smallest trouble, *ad infinitum*. To the axle a piece of log-line was fastened daily, on which the conductor tied a knot for every mile, that is to say, every time he reversed the wheel.

This perambulator was for military operations, and answered well; it was first introduced in India, about the year 1778. The invention of this species of perambulator is, however, due to Mr. Tugwell, who, so far back as the year 1768, (if I err not), had completed one similar to that, in the collection of the Bath and West of England Society.

In surveying a road with a perambulator, the person who conducts it, should always make as straight from the center of one turn, to the center of the next that may be in sight, as possible; just as if a line were drawn; by this means he will, in the long run, remedy the fault of the perambulator, especially if the wheel is small, as in the dial kind. It is well known that the fore wheel of a coach enters hollows in the road, more than the hind wheel, which is larger; consequently, the former must

touch more ground, and turn round oftener; and all wheels partake of this disadvantage, more or less, as they follow the road in descents, which not being very long, are passed over when measuring with a chain. It is obvious, that by going straight, from center to center, of turnings, as from A to B, B to C, C to D, &c. this is in a certain degree balanced, and gives a more true reckoning, than if the center of the road were followed all the way, according to the dotted line, for the wheel goes over too much ground, while the straight line cuts off about an equal proportion from the real measurement. See Fig. 5. Plate 9.

When a road is to be carried straight from one point to another, let the remote points be distinguished by a conspicuous flag, &c.; and let an assistant, being provided with several wands, come to within a hundred, or two, of yards of the surveyor's station; whence, by means of a sight-instrument, directed to the remote flag, he may be guided in pitching the first and second wands exactly in the line of sight; after which, if the assistant has any judgment, he can place the rest of the intermediate wands, by looking towards the surveyor, and walking backwards towards the remote point. The surveyor should, however, occasionally take an observation, lest from carelessness, or insufficiency of judgment, the assistant might deviate from the right line.

SURVEYING ON EMERGENCY.

PLATE 20.

It sometimes happens that a large estate is to be surveyed in a very short time; in such case, proceed as follows:

Draw a base-line, AB, at the narrowest, or most favourable side of the estate, and from that set off as many perpendiculars, CD, CD, CD, running through the lands, as may be requisite; observing, that in such case, it is best to run one through every series of fields, so that no part may escape notice. Begin at C, which is opposite the middle of the right hand field, proceeding in a straight line, notwithstanding hedges, &c. (through which your chain, or line, must be pushed, by means of a pole, as before directed); and, whenever you arrive at right angles with any corner, or the center of any curve, &c. draw an offset from your main line, noting down its length, the distance on the main line, and the field in which it occurs. Preserve this plan until you arrive at the boundary EF, which is the parallel to your base-line; then, (as you began on the right), proceed measuring to your left, parallel, if practicable, with your base. When you are in the direction of another series of fields, turn again to your left, so as to return to your base in a second line, parallel with the first main line. Again, measure off to every prominent feature, never omitting to notice the corners of fields in particular, and marking down the gates according to judgment, as also the places, and extents of watering-pools; houses, barns, &c. should likewise be mentioned; and duly placed in your plan.

Being returned to your base-line, measure off to your

left, until you come opposite to a third series of inclosures; proceed through them, backwards and forwards, from base to parallel, and from parallel to base, making so many main lines as the breadth of the property may require, as may be seen in the following plan, in which it appears that four main lines are sufficient for the survey of the several ranges, or series of inclosures, whereof it is composed.

The figure *one*, on the base, A B, shews the first outset from C to D; the figure *two*, on the parallel, E F, shews the return from D to C; then *three*, on the base, shews the second outset from C, on the base-line, to D, on the parallel; where *four* shews the return; and so on, as often as there may be occasion. If the hurry is great, the base-line may be dispensed with, provided great care is taken; else the base-line should go the whole breadth of the property, and be clear of all the projecting points, so as to afford a clear sight all along it.

Now, as all the main lines are regularly measured, as to length, and their respective distances are also ascertained; and they being at right angles with the base, and mutually parallel, the quantity of land contained within them must be established. The offsets being taken to the several corners, or projections, and the irregular curves, &c. being examined by the same means, (if of consequence enough to demand such minuteness), not only will each field be accurately delineated, but, at the same time, each will be so quartered, as to give immediate opportunity, according to the rules before laid down, for treating them as trapezia, whence their several contents may be known.

In walking through each field, its designation should be recorded; the timber may be generally spoken of, whether oaks, elms, &c. also their state of growth, and average number. But the surveyor may, while the off-

sets are taking, cast his eye around, and count such as are of a marketable size. Grass and arable lands should be distinguished, together with a thousand particulars, that must strike every person on the spot; and which, if they escape his notice there, he would probably treat with the same indifference, if detailed in this place. I am loth to burthen the volume with such matters; and, from the same principle, I avoid entering into all those prolix arithmetical calculations, which the man of common abilities cannot require; and which, to the idiot, would be as useless as waste paper!

The above mode of surveying, as it is ludicrously termed, "*on horseback*," has been long practised. It is said to have been invented by a person of the name of *Mickleham*, who, so far back as the year 1687, published a treatise on surveying with the cross-staff, which is equal to this purpose, if the lands are level. I see it again in John Hammond's *Practical Surveyor*, printed by T. Heath, 1725, but used with a theodolite; and I observe it in Mr. Davis's work, before quoted, with some improvements of about thirty years standing, and some added superfluities, such as absolutely do away the word *expedition*, and bring the survey to the common course of precise measurement.

TO ENLARGE OR DIMINISH A PLAN.

THE surest and best mode is to do the whole over again, by means of a new scale, proportioned to the change of size.

A quick method for copying, on a different scale, is to

divide the original into any number of equal squares, or parallelograms, by a faint pencil-line. Divide the paper, or space on which it is to be copied, into an equal number of similar squares, or parallelograms, and trace in the plan by your eye. The exterior proportions of the original, and of the copy, must be exactly the same, else the divisions will not correspond, and the plans will not be similar.

The following figures, 2 and 3, Plate 20, will shew the manner in which this operation, (which is extremely common among engravers and other artists), is effected.

Fig. 4, in the same plate, shews how to increase the length, by diminishing the breadth, or *vice versa*.



LEVELLING LANDS FOR IRRIGATION.

THIS is a most important branch of the surveyor's business, for it includes not only the laying down the levels, one above another, something like a flight of very broad, shallow stairs, but the conducting of a stream, and the laying out the drains in such way as to carry the water, with an even flow, over every part of the soil. The former mode I shall treat under the head of

SIMPLE IRRIGATION.

It would be impossible to enter fully into this matter, without devoting a volume thereto; I can only state, in

general terms, that the main conduit, A A, Fig. 1, Plate 21, should lead into an open reservoir, B, railed in, so as to keep cattle from making overflows; and that the reservoir should, especially if the stream is small, be of sufficient capacity to serve for one complete irrigation. It should be at that part of the land which might be high enough to carry the whole above the surface to be watered. From the reservoir should be a small channel, C C, into which the water should flow, at pleasure, by means of a flood-gate, or sluice. This channel should, if practicable, lead through the center of the irrigation, having a small sluice, D, D, D, D, at each level, so as to keep the water in to any extent. At the foot of each level, E E, E E, E E, in a line with the upper side of the sluices D, D, D, D, a small drain should be cut, of which the excavation, as well as that of the main channel, would apply to filling up hollows; this drain serves to receive the water, before its rise to that height, which may cause it to flow over its bank, into the next lower level; and, when all that would rise to the due height may have passed over, then, the sluice being opened, will cause the residue of the water in the small drains to follow into the next level.

The first turf cut from the surface of the small drains, may be laid, the grass downwards, between them and the next level, so as to keep the water in, to a height, before it runs over; but this need only be done when there is much declivity.

By this it will be seen, that each level is watered in succession, (the water drawing off from the level O, going next to the level P, and so forth); and that, ultimately, the water goes off by the said channel C C. The whole operation may be conducted by any commonly intelligent person; but, as the least mistake, in regard to apparent levels, may cause much disappointment, delay,

and useless expence, it is best always to ascertain that point by professional examination.

There is another species of irrigation, that requires some calculation, viz. the watering of meadows that lay below the level of a stream, passing either through, or near them. In these cases, the surface is frequently somewhat level, but often full of hollows, and, by laying far below the stream, becomes subject to long and superabundant overflows. The best remedy is to bank out the river, leaving sluices at proper places; then dividing the land, by broad, deep drains, into equal portions, let the excavated earth be applied to raise and level the square compartments thus made. If this be done with judgment, the surveyor will often gain great credit by rescuing lands from perpetual inundation; for, besides raising the surface of the meadow, he will thus form boundaries, such as cattle will not attempt to force through. This is not to be done in all situations; but, where practicable, should always be pointed out by the surveyor, who should calculate with great accuracy how far the lands may be preserved from the inundations, by applying the excavation from the drains to that purpose; in fact, his drains ought to be cut deep enough to afford a substantial rise to the soil, else he will create a heavy expence, without affording proportional benefits.

I shall now proceed to the second mode of watering, under the head of

COMPOUND IRRIGATION.

This particularly relates to lands which are so irregular as not to admit the practice of *Simple Irrigation*. We often find pastures laying in various declivities, presenting several fronts, as it were, and requiring some

judgment to conduct the water, so that the whole of the land shall derive equal benefit. One example will suffice, to convey a full idea of the intention of this particular, and may serve to shew how easily the most broken surfaces may be duly watered, provided the surveyor will take pains to ascertain the exact level on the summit, or brow, of each inclined plane, and so carry his principal drains, that every channel may, in its turn, receive a supply of water adequate to flooding the proportion of soil between it and the next lower level.

In Fig. 2, Plate 21, the main conduit, e, e, is supposed to enter at a, by means of a small gate, or sluice; the water is to have only a slight descent between a and b, where it is kept up by another sluice, until it rises into all the little channels, d, d, d, d, which are stopped at their other ends, in the line h, h, so that it may flow equally to the right and left, over the land, which is laid out in ridges, or pitched, like the tops of houses, and occasions the water to be received into the bottoms, or gutters, k, k, k, k, which lay between the ridges, whence it is conducted into the second, or next lower course of drains, i, i, i, i, laid out in a similar manner, whence the water flows, as before, right and left, over the sloped sides, into the gutters, m, m, m, m, which convey it in another lower level, if thought proper, and thus, into any number of channels the stream may be capable of supplying; but as, to carry the fluid through any numerous succession of levels, in this mode the upper drains would require to be kept very full, and endanger the loss of the finer parts of the soil, owing to the force of the current, it is better to lead the main drains in a meandering course through the land, (and, indeed, unless the surface be very regular, this will be indispensable), causing it to flow gently into the return, f f, which has another sluice at c, and keeps the water up into the third levels, which

are watered in a variety of ways, according as the land may lay. Thus the water is received at the drain n, and flows, right and left, into o and g, which convey it into the third bend of the stream. This is done, because the land naturally has a ridge, or back, in the direction of the drain n. Opposite the arrow, which shews the course of the current, the soil may be watered in two ways, namely, by flowing out of the main conduit, or by receiving its supply from the drain o, (which in such case must be stopped below); and thus the several drains, p, p, p, p, may get their supply, emptying into the drain r, r. Now the residue near the lower sluice, we suppose to be an evenly declining plane; therefore the water is allowed to flow from the main conduit, and being broke by the several cross drains, as in *simple irrigation*, each having a small sluice, as at s, s, s, s, it will pass in an easy succession, into the third parallel of the conduit at g, g, g, g.

The profile, or section, given at the bottom of the plate, shews how the water is conducted, first from the main drain, v, into the drain w, on the top of the ridge, whence, falling into the gutters, it proceeds into the second level drain, x, which supplies the second course of ridges, as at y; when coming still lower into the drain z, it glides over the inclined plain u, laying in a different direction from the two courses of ridges.

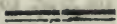
So much has been written on this subject, that little room is left for instruction on this head. I cannot, however, forbear from remarking, that by means of the process laid down in this treatise, the farmer may take his own levels, and conduct his own drains, without the very expensive aid of a *water-layer*, who generally is so much hurried, as not to be able to give due attendance to the work as it proceeds.

Suffice it to say, that a fall of half an inch, or perhaps

less, to every foot of breadth in the ridge, will carry off the water full fast enough on light thin soils, which would be apt to wash away, were the fall greater: heavy soils will bear rather more; though I should not recommend it. The quantity of water flowing over the upper range of ridges, may be as much as the soil will bear, without washing or ripping the sod: two things always to be avoided. As some of the water will necessarily be lost by absorption, which is indeed the principal intention, so each succeeding level of ridges will be more and more scantily supplied. This will often, of itself, determine where a new course of supply must take place, by a turn in the stream, and which generally, in loose soils, is needful at every third or fourth level.

As to the time allowed for the flow of water, that must depend entirely on circumstances: the supply is a principal consideration; for where the source is scanty, and rain expected, less should be used; whereas an ample stream, and dry weather, dictate an abundant washing, even until the soil may be completely saturated.

Observe, that where the water is turned on, a person conversant in the business should be at hand, to watch, in case of the drains choaking, to open the several sluices in succession, as may be necessary.



DRAINING OF LAND.

THE sides of hills generally abound in springs, which, if kept up by clay, will break out in various parts, and make the soil "*springy*, or *spouty*," as it is termed. To remedy this, a surveyor is generally employed, who

should direct the dimensions and courses of drains intended to carry off the moisture. Some open drains may be necessary, to which the concealed drains ought to incline, and discharge themselves. This kind of under-draining, which has proved of the highest importance to agriculture, should never be carried straight down a hill, but obliquely: the drains may be from fifteen, to thirty feet asunder, according as the land may require; their depth must vary with circumstances; generally, twenty inches deep, and six or eight in breadth, suffice; they are filled with stones or stubble, furze, &c. and covered with soil, to the ordinary depth of ploughing.

But, although these drains last for an immense time, and carry off astonishing quantities of water, they are not to be wholly relied on. A well, of about four feet in diameter, should be sunk near the place principally infested by the springs, and should be carried down, until a bed of sand, chalk, &c. may be found. Even boring to a great depth, with the common boring auger, of three inches diameter, has been found effectual, where it could get through the stratum of clay, so as to give the water what it seeks, viz. a vent downwards. This was, at first, kept a profound secret by Elkington and others, who made immense sums by their mystery!

These *springy* soils result from cavities in the hills, which contain large quantities of water, and, often overflowing, the fluid finds its way to the surface, in various directions, generally about one-third up the ascent. I know one instance, where not only the mischief was prevented, but the water turned to purpose, it being conveyed to two pools, that had always a plentiful supply for cattle. This the surveyor should have in mind, when employed in laying out, or in improving landed property.

MILL-STREAMS.

IF the water-wheel be properly constructed, a very small quantity of water will suffice to turn it. The principal point is, to obtain a proper height of fall, which is generally done by means of a weir, thrown across the stream, holding the water up to the required level. The surveyor, having to conduct a run to a mill, must be tolerably exact in his levels, and should rather err in bringing too much, than too little, as a sluice will always remedy the former evil, while the latter is not so easily got over. The generality of undershot mills require from four to five feet of fall; to produce which, it is sometimes necessary to get the water, through a small channel, from a great distance up the stream, as a weir could not always be made at the mill, either from the litigation of other parties, or from the probability of inundating large quantities of low land. In this case, the water must be conveyed on a bank, raised to a suitable height; and this too must often be done for supplying a reservoir for irrigation; for which weirs would prove, in the first instance, very expensive, or possibly could not be constructed, for various reasons.

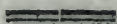
MINES, &c.

A SURVEYOR is sometimes employed about coal, marl, ores, &c. especially where they are to be worked by engines. These, properly, appertain to the engineer; but, it often happens that an expert surveyor, by a knowledge

of this branch, derives great profit and credit. He should, therefore, understand the principles and mechanism of the steam-engine, and of all such contrivances as are, generally, applicable either to raising the mineral, or to the discharge of water, which, in almost every mine, flows in great quantities. Of these notice will be taken in a subsequent work, to which this volume is, in a manner, preparatory. At this time it is sufficient to say, that the surveyor should carefully keep the proprietor within the bounds of his own estate; as, from want of caution on this head, much litigation and damage might arise.

As the work proceeds in the mine, its direction should be carefully ascertained by means of a compass; and, if the lands of another be contiguous to the field, &c. wherein the shaft has been sunk, the surveyor should, from time to time, observe the progress below, and shew the extent of excavation, by driving stakes in the field, exactly over the track pursuing by the miners.

With regard to valuing coal, or marl-pits, the surveyor must consider them as solids, and having ascertained the average depth, should multiply that by their known, or supposed superficial extent, whereby a tolerably just appreciation may be collected.



TO MEASURE GROWING TIMBER.

PROVIDE a slight pole, about six feet in length, with a spike at bottom, to fix in the ground; and across the head of the pole, screw on a piece of wood, about two feet long, two inches broad, and one in thickness, mak-

ing an angle of *exactly* forty-five degrees from the perpendicular. See Fig. 1. Plate 22.

Fix your pole opposite that side of a tree which affords the best view of the summit of its main stem; that is, up to that part which either can square, or may be merchantable timber: a plumb should be affixed at the side of the pole, or standard, to set it exactly perpendicular.

Proceed to such a distance from the tree, as to you may seem equal to its height, looking over the angular board, and removing your standard nearer, or otherwise, to the tree, until the sight taken, cuts that part of the tree, up to which you calculate as timber. Measure the distance from the foot of the standard; that added to the height of your standard, at the center of the diagonal, is the length of the timber.

This mode I hold to be of my own invention; it is extremely simple and certain; for it is clear, that the line drawn at an angle of forty-five degrees, will give a perpendicular equal to the base; but, as that base must be produced from the level of the diagonal's center to the ground, to include the whole of the lower part of the tree, so the length of the pole must be added, as will be confessed, on a view of figure 2; else the line of sight must be continued, as marked by the dotted line to A, which will amount to the same thing, as the measurement from A, to the tree, at B, will be its true height.

The altitude thus obtained, the girth may be easily taken up to the height of rather more than six feet, by a cord; one quarter of the girth is considered as the square. I have already spoken of this in Prob. XXXIII. which teaches the inscribing a square in a circle.

With regard to the girth above, it must, generally, be taken at a guess, which, by practice, will frequently give a fair medium. I know not how any rule can be found

for what is often so irregular, and so capricious, as a tree in its growth. We must judge by appearance; but we may gain much information, by taking the measure of a few trees, after felling, whereby our judgment may be corrected, better than by all the diversified computations that ornament whole pages of many elaborate publications!

I cannot see any difficulty in taking the exact height of a tree: if the standard triangle is used, it matters not how far, or how near the spot may be from which the top of the timber may be seen; for if the distance between the standard and the root of the tree be tolerably level, and can be measured, the angle made by a sight, from the hypotenuse to the end of the timber, will shew its altitude.

This will give the measurement of trees that grow upright; such as incline but little, may have a small allowance made in addition to the height, as thus taken; which, in such cases, should, properly, be from the side, so that the tree may appear to be falling across the line of sight.

If a tree leans very much, measure the angle at which it reclines; ascertain, as nearly as is practicable, the perpendicular, from the timber-end, to the ground; then, as you have one side, viz. the distance between the tree and the perpendicular, and, as you have likewise ascertained two angles, the height will be as the hypotenuse of a right-angled triangle, thus formed. See Fig. 3.

From a to c will be the real length of the tree, though its height will appear to be only from a to b.

OF PLANNING ON PAPER.

THE first thing is, to ascertain that your paper is large enough to contain the whole of your plan, according to the scale you adopt for the occasion. I have already, in Problem VII, shewn how to make a scale, of any extent, for your own use; but, where your business is extensive, it is better to have a three-foot flat ruler made, expressly for your purpose, with various sized scales, lines of chord, &c. all on one side. You should also be provided with a pair, or two, of compasses suitable to your work, such as are generally sold in long cases, with sliding tops: I also recommend Marquois's parallel scale, as it not only makes a parallel at pleasure, but a perpendicular thereto, if required; it consists of two pieces; one in form of a protractor, or flat scale, the other a right-angled triangle, which slides along the rule in any direction that may be given.

Further, it has the peculiar advantage of forming the parallel either obliquely from the original line, (that is, more to the right, or left), or in a vertical line underneath. I know not of any instrument so well suited to the delineation of flights of stairs, &c. as the above positions of the triangle, as it slides along the edge of the scale will exhibit. (See Figs. 4 and 5. Plate 22.) I should suggest an improvement on the triangle, by making its rectangle a point, whence, on the hypotenuse, a scale of degrees might be graduated; by which means, parallel, or similar angles, might be made in the most expeditious manner, from any given point on the line directing its course.

For very large works, the parallel rules that move on rollers, with ivory scales on their edges, are certainly very

convenient, but they demand great attention, and steadiness of hand; indeed, they should almost be left to themselves, for the least thing, even a roughness in the paper, will often change their direction. On some of this kind, the wheels are graduated, to shew the number of lines, or eighths of inches, the ruler has passed, from that line to which it was applied in the first instance.

In every other respect, the ordinary cases of instruments are sufficient; but those which contain the instruments flat, like cases of surgeon's tools, are far preferable to such as have them upright; the latter are extremely injurious to the points; and, as the instruments will sometimes, especially after much motion in the pocket, &c. stick in their places, they are further subject to injury, from the force necessary to liberate them.

The paper should be large enough to leave a fair margin around your plan, and should be carefully selected, especially if your work is to be coloured; for, it often happens, for want of due precaution in this particular, that what has taken much time and care to delineate, has been completely spoiled in the colouring, by the paper being either greasy, or damaged. Before you begin your work, take the paper to the window, or to any strong light, and see there be no oily specks, nor brown, nor mildew blotches, nor, (which is very common), some parts thinner than others, for all such will, assuredly, disfigure the plan.

Good wove paper should appear, when held to the light, smooth and even, without any kind of flaws; it should be something like very thin vellum, and the water-mark should be very faint. To know whether it will stand the colouring, wet a corner of it slightly; if it is good, or what is called *hard* paper, the moisture will remain a long time on the surface, before it penetrates into the size; but, if the moisture sinks rapidly, or shews

as though oil had dropped suddenly on it, such paper is only fit for printing, being what is called *soft* paper.

If you cannot get paper that stands colouring well, you must dissolve some alum, about four ounces, and boil it in a quart of water, till the alum is dissolved. If your water is not very pure, throw in the whites of one or two eggs, as the water begins to simmer; let it be skimmed, and, when cold, filter it through a funnel, lined with clean blotting paper, made to fit within it, so that the water must be filtered through the pores of the paper.

Your paper should be dabbed with the above, by means of a sponge, free from harshness, and by all means avoiding to rub the surface, as, by so doing, it would acquire a roughness never to be got rid of: the paper should be moistened only so as to appear damp, and not too much soaked, which would make it puff, and appear uneven, unless the edges were previously confined down by paste, &c. If the alum-water is properly managed, it will give great firmness, and make the paper resist the imperfections in its manufacture in a wonderful manner! Sometimes it is necessary to sponge the paper twice, or even thrice; but it must be completely dry between each operation.

Your camel-hair pencils ought to be full, long, and well pointed; the best way to procure them is, from the original makers, who either keep small shops, wherein they vend every kind of brush used by artists, (and nothing else), or, they may always be heard of among people in the turnery line.

Your colours are principally Indian Ink and Bistre. Of the former, you may get either black or brown tints, by selection from large quantities. I do not know any place where it can be found so genuine, or so smooth, as at Mr. Newman's, in Soho-square, whose cake-colours are peculiarly transparent! It is useless to burthen yourself

with a great number of colours; the following may answer every purpose:

REDS.

Carmine, lake, vermillion, red lead, Indian red,

BROWNS.

Umber, bistre.

YELLOWS.

Gall-stones, gamboge, king's yellow.

BLUES.

Ultramarine, indigo, verditer, and in this we may include neutral tint, which is excellent for distances and back grounds, and is best ready prepared.

GREENS.

Verditer, verdigris, and sap-green.

BLACK,

Indian ink, ivory black.

WHITE

Is a body-colour, generally made of white lead. That which is called flake-white, is the best; but this colour does not stand. The shells of eggs, burnt to a clear white, and ground with a little gum-water, make the best white, in regard to preserving its purity. The white lead, when used in water-colours, turns ultimately to a dirty black.

Gum-water is easily prepared, as follows: select from a large quantity, say from a pound, of choice gum-arabic, as much as you can find that is of a pure white, very brittle, and easy to dissolve. Suppose you can collect two ounces, which is as much as, probably, will be found to answer your purpose; put it to steep in a quart of boil-

ing water; stir it every two, or three, hours with a clean stick, or a spoon, until it is dissolved; then, over the surface, spread a piece of clean blotting-paper, rather larger than the surface of your vessel; this will sink gradually, and carry with it all the gross particles that may be in the water.

If you find the above preparation adhesive, so as to be clammy in mixing with your colours, or, that it makes them very glossy, put a little more water, filtering it through some clean blotting-paper. When you apply your colours, a little of this gum-water, mixed with the water you use for moistening them, will prove highly useful in binding them, so as to resist damps, and will, besides, aid the alum in preventing them from running.

Let your plans, unless ordered to the contrary, be very neatly executed, in plain washings of India ink, mixed with a little bistre, for the lighter tints; or, eventually, in describing woods and hedges, with a small portion of sap-green; waters may have a dash of clear blue, but very faintly laid on. This, if sparingly managed, so as merely to distinguish the objects, without *blazoning* them, will give an appearance to your work, such as will shew you possess delicacy and taste, and that you are not to be led astray by any old customs newly revived, (as before remarked), to daub your margin with hieroglyphics, coats of arms, and all the trash that bespeak a vulgar habit, and that should seem intended as a lure, to call off attention from the more interesting points. These are like the tricks of mountebanks, or jugglers, who amuse their audiences with nonsensical gambols, while their confederates are carrying on their deceptions, or indulging their curiosity, by examining whether the gaping crowd have better purses than brains!

Be assured, that unless your consumption of colours be very great indeed, (equal to that of a vendor), it will

never answer your purpose to prepare them. The apparatus is not devoid of expence; the time they occupy, not inconsiderable; the filth and rubbish they create, very unpleasant; and, after all, probably, you will not equal the colours to be had at any respectable makers. Some make one colour excellently, but do not succeed so well with others; this proceeds entirely from a want, either of chemical knowledge, or of chemical apparatus. On the whole, Newman's colours are the best, as is now universally acknowledged by the numerous artists, as well as families of the first distinction, and others, who have, for many years, given them that preference their superiority merits! Next to them, I recommend the colours prepared by Warner, in Piccadilly. What are sold at the shops of print-sellers, &c. &c. are, generally, manufactured for them, *wholesale*, by oilmen, who are neither very careful, nor very intelligent, in their preparation.

OF PLANNING FROM THE PAPER.

I SHALL conclude this treatise, with instructing the student in the manner of setting off the plan of a house, &c. from the paper, by means of the standard triangle.

It must be understood, that either every part is marked as to dimensions; or that a scale is given, whereby each may be respectively ascertained.

In laying down the plan of a house, in general the site of its front is known; therefore we will take that as a leading feature, and suppose the whole face to be in a line, no part projecting or retiring.

Let the center of the front be ascertained, and let a

line be stretched for its whole extent, and set off the thickness of the foundation, allowing one foot increase of width, for every three feet in thickness, and again for every three feet in depth; thus, if the wall is to be two feet thick, the foundation should be eight inches broader, equally dividing them, four in front, and four within. Then, if the soil requires a foundation of more than a yard in depth, what may be below that level, must be breadthened one inch for every three of the incumbent thickness; therefore, in this case, an increase of eleven inches, and one-third, should take place, and so on to any depth.

This is properly the architect's business; but as it often becomes a question to the surveyor, whether a house can be built on a certain space, it is proper he should be reminded of the increase of width below, to be occupied by the supporting masonry; for when laying down the plan from paper, allowance must be made for the increase of substance just described.

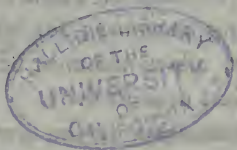
Having ascertained the situation of the first wall, A B, let its breadth be set off, and by means of the standard triangle, draw the two ends, A C, B D, perpendicular to the front; continue them to their intended length, and draw the line of the back, C D, parallel to the front. Set off the thickness of the party-walls, and subdivide the internal area, according to your plan, allowing duly for the several partitions, and seeing that every thing stands at a proper angle.

After the excavation for the foundation has been made, according to the spaces marked out for that purpose, as shewn by the dotted line, the surveyor should lay down the utmost thickness of the walls, always remembering that the superior part of the foundation, as it rises to the level of the ground, or to wherever the ground-floor is to be laid, will guide as to the dimensions of the

apartments; for there the wall narrows to the substance it is to preserve above. See Fig. 6. Plate 22.

The courts, gardens, out-offices, must all be put into their proper places, and their bounds duly described from the plan, by means of a line, or chain, guided by sights from the standard triangle.

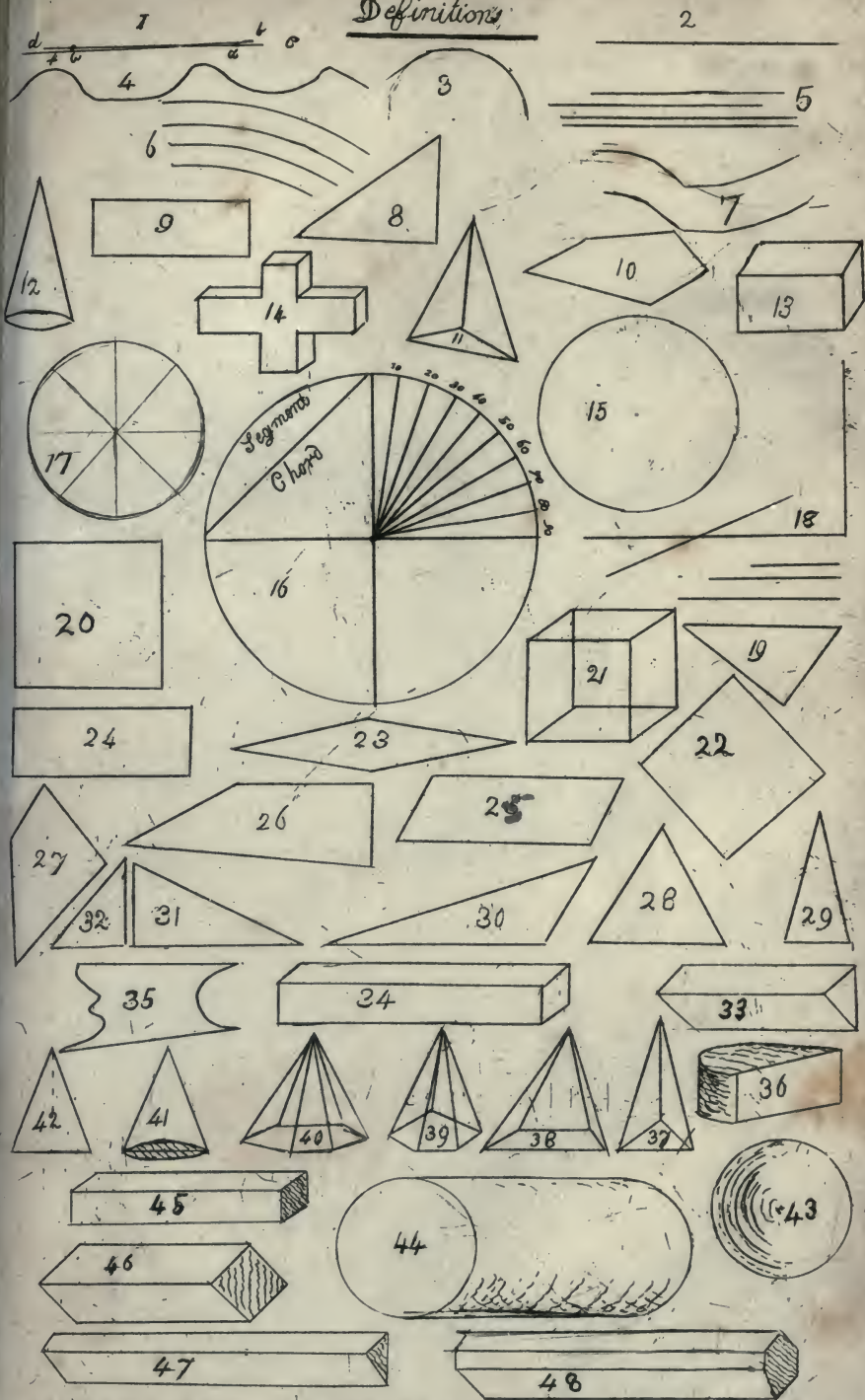
Lands may be laid out on a large scale by the standard triangle, by dividing the several fields, &c. in the plan, as in a survey, and setting off their angles from it, by similar angles or figures, formed by means of sights and measurements; in fact, by inverting the whole process. If the foregoing pages are properly understood, the surveyor cannot be at a loss on this subject.



THE END.

Definitions

2







Problems

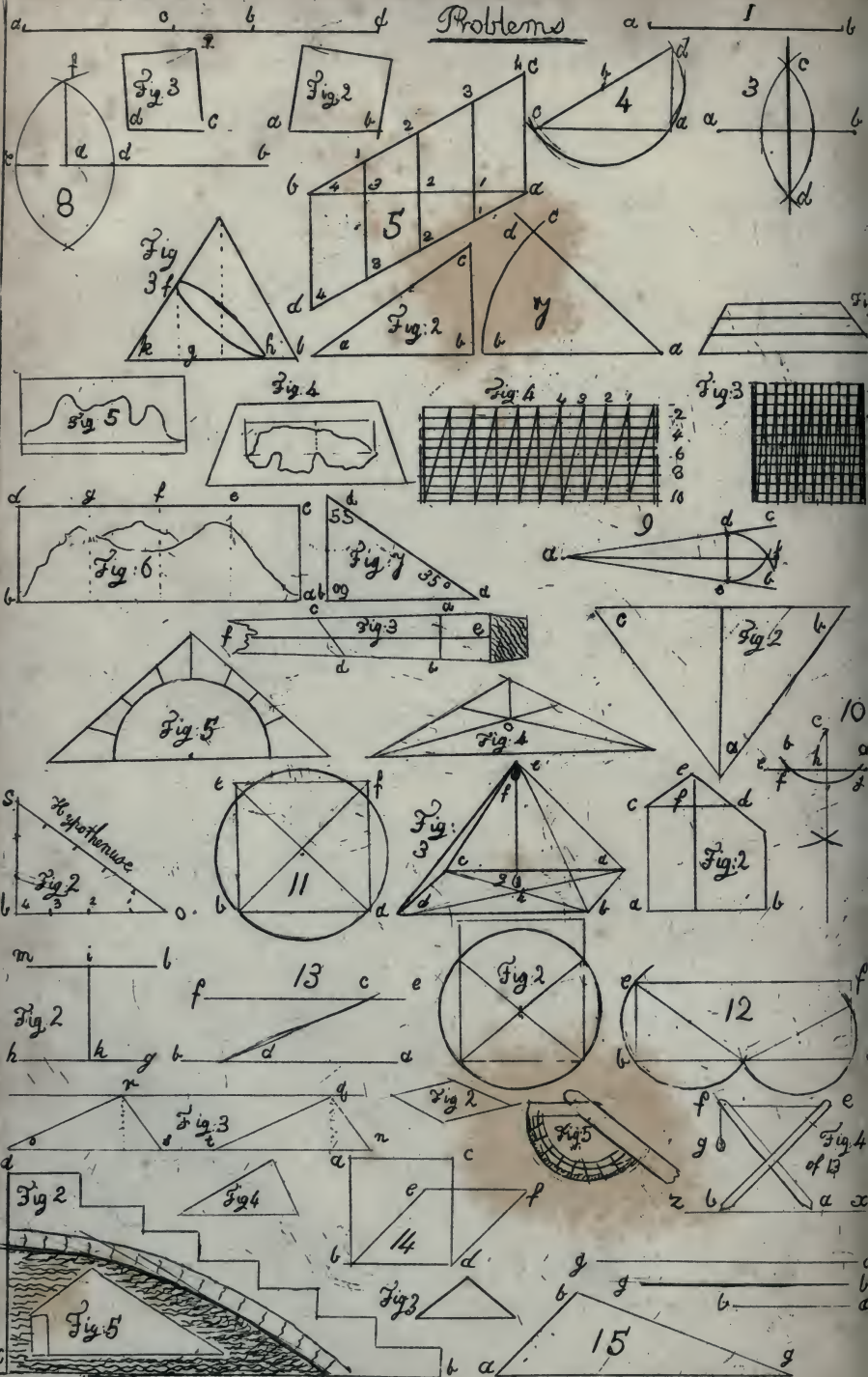


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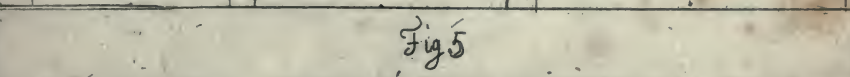
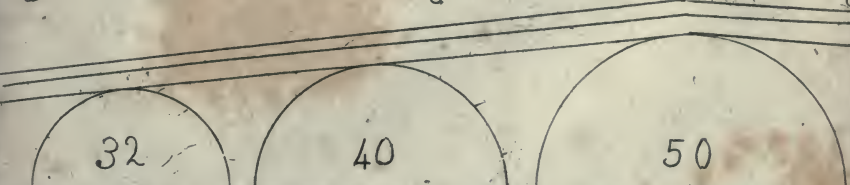
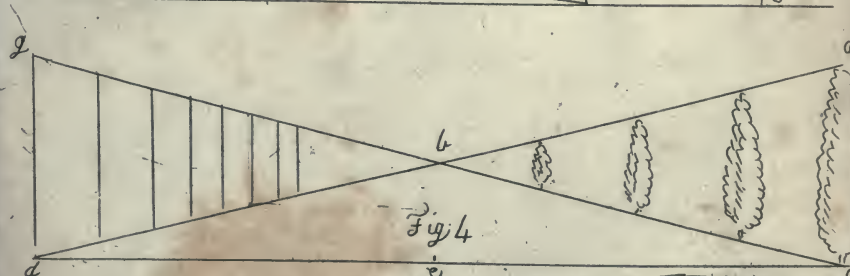
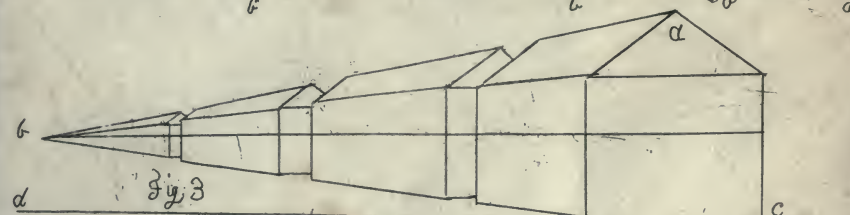
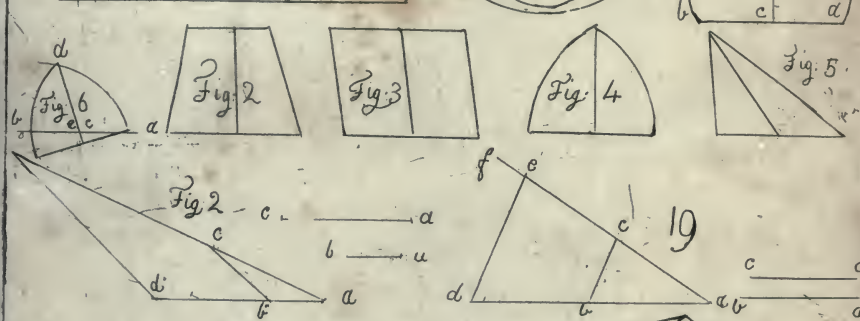
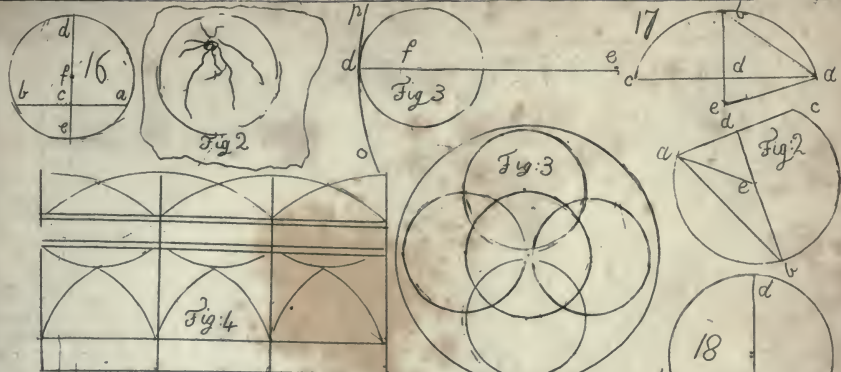


Fig 5





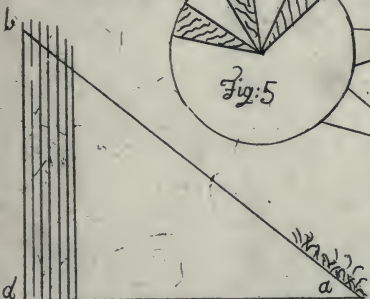
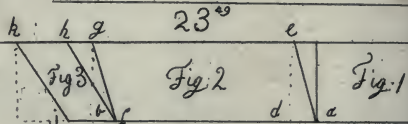
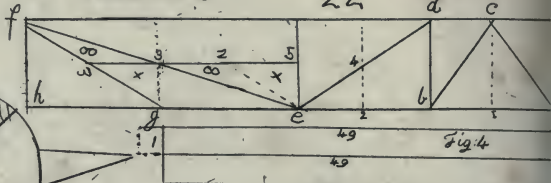
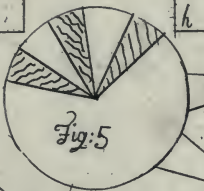
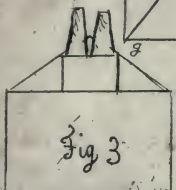
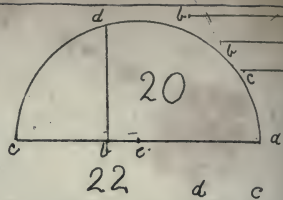
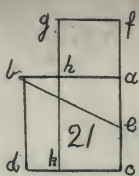
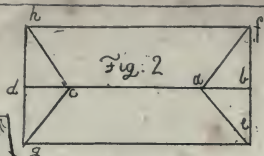
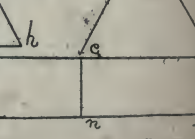
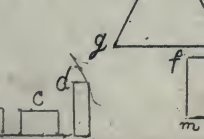
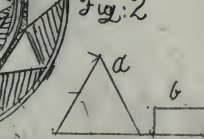
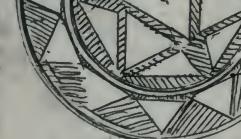
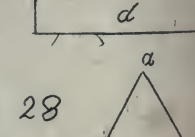
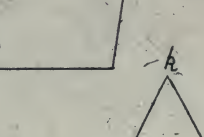
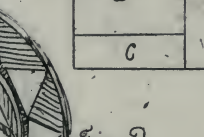
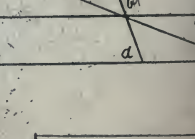
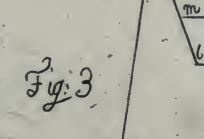
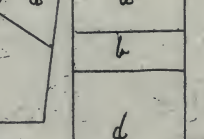
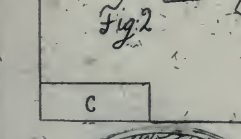
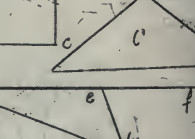
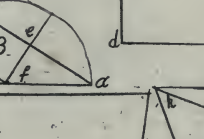
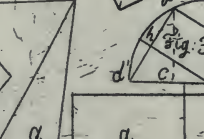
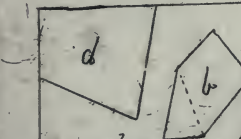
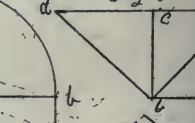
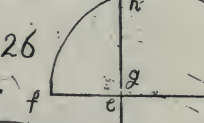
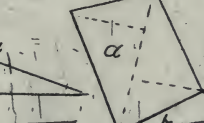
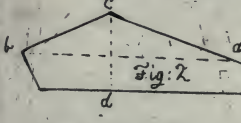
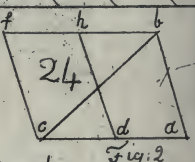
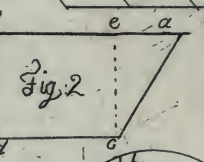
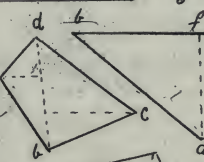
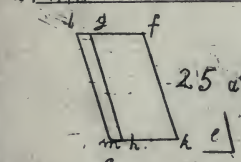
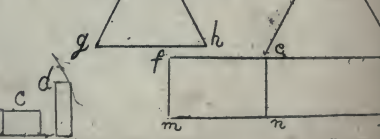
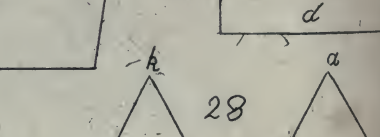
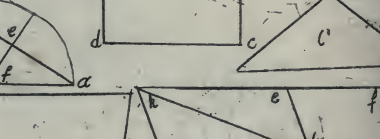
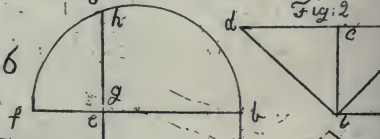
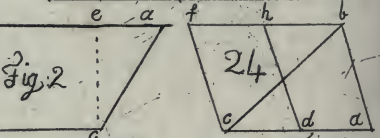
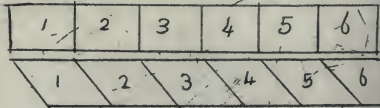


Fig. 3

Fig. 2



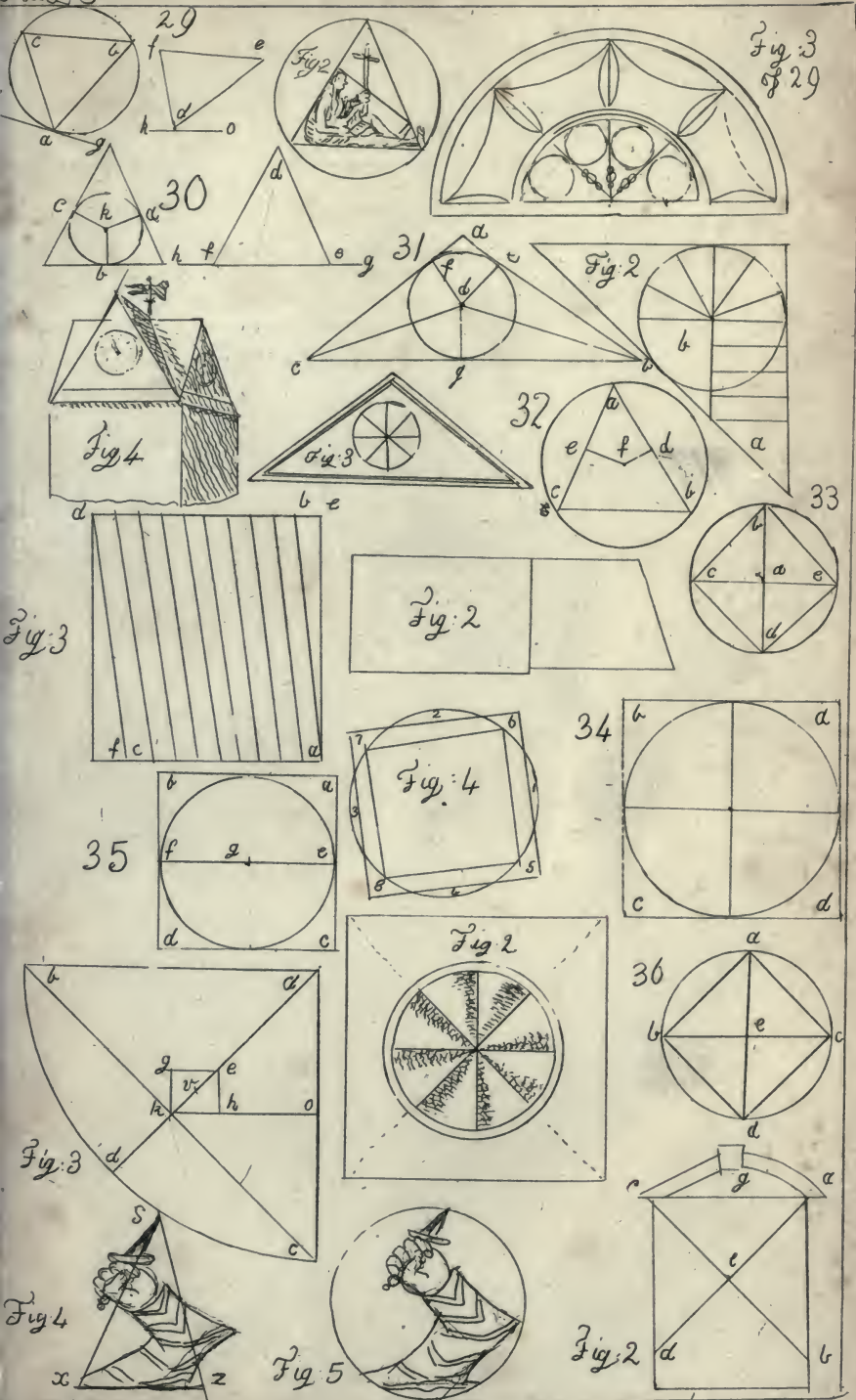
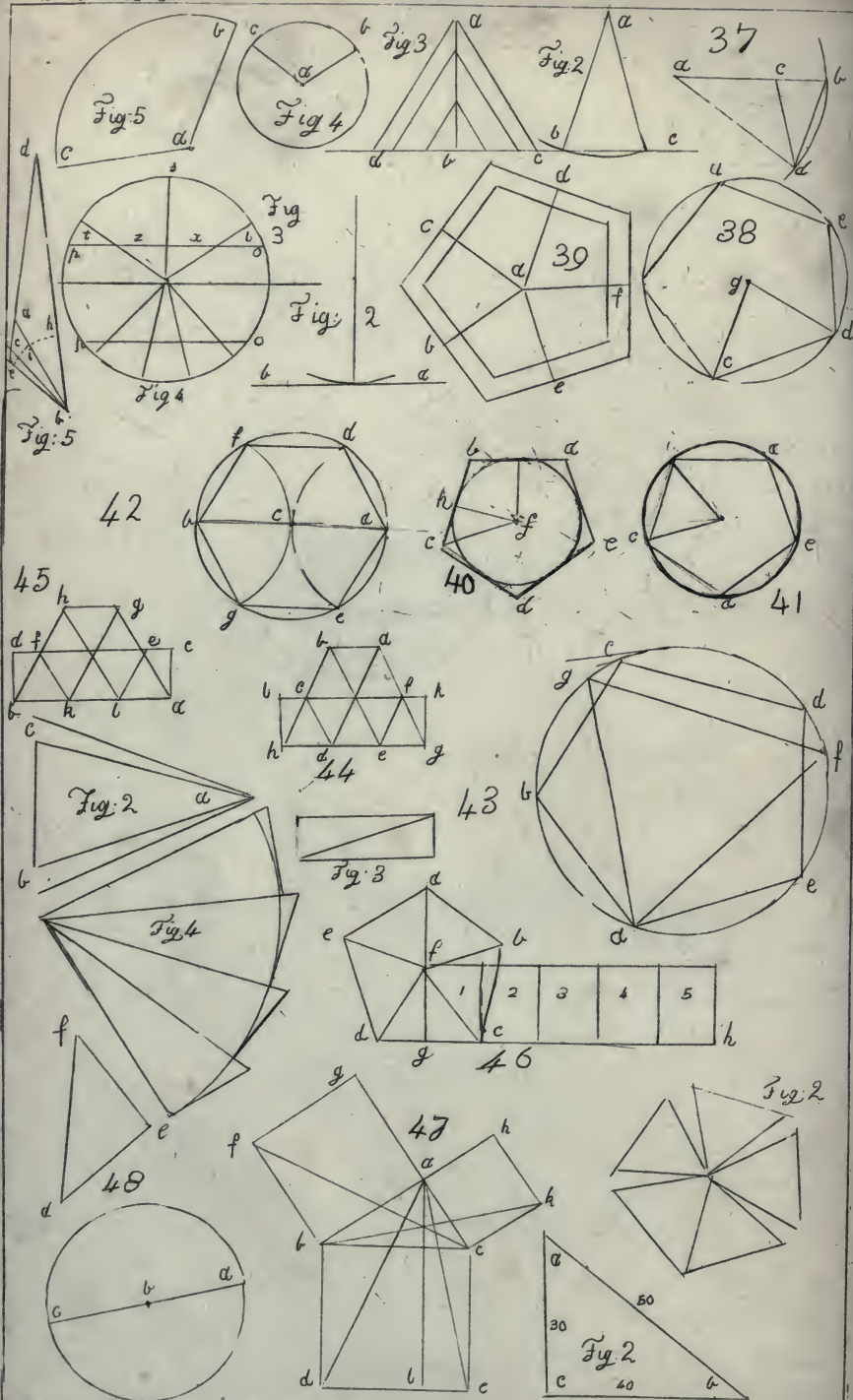






Plate 6



Problems

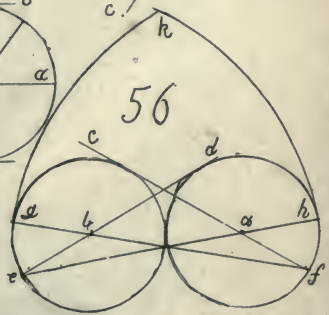
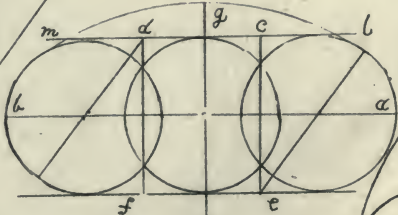
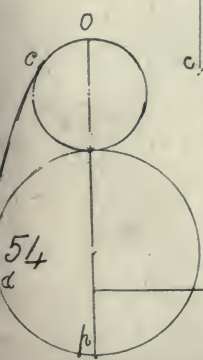
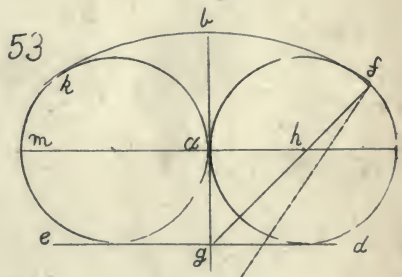
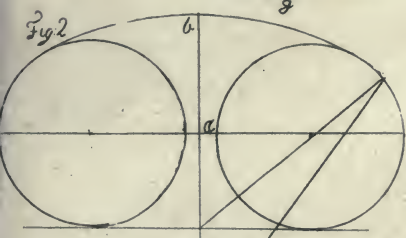
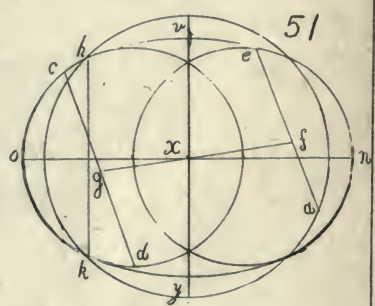
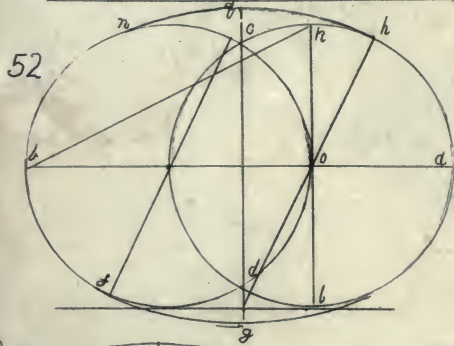
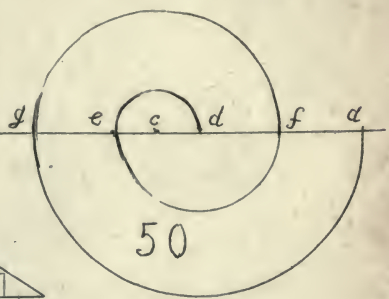
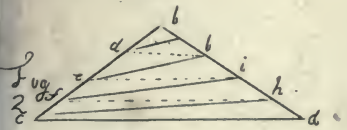
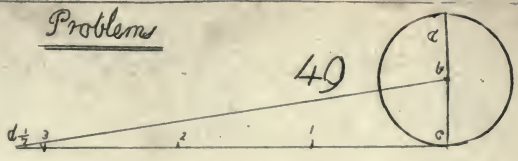
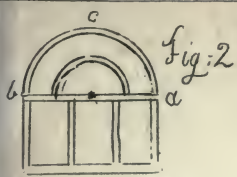




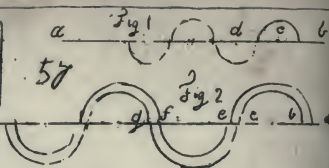




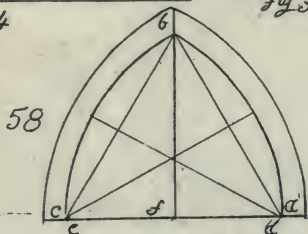
Fig. 4



Fig. 3



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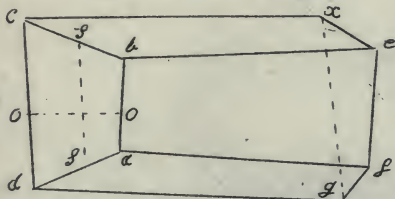


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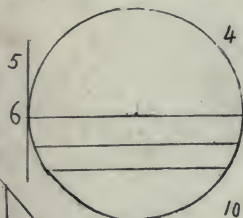
Fig. 5



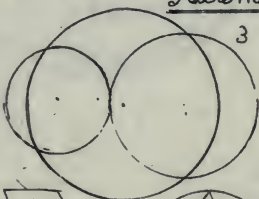
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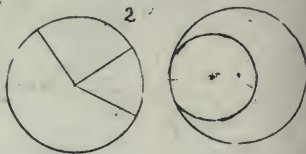
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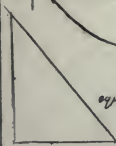
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2



equal



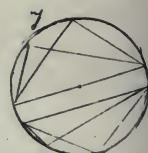
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8



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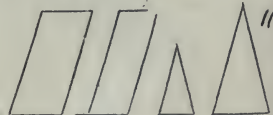
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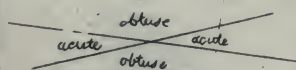
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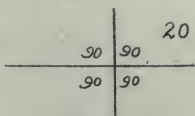
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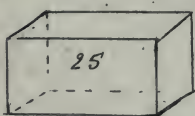
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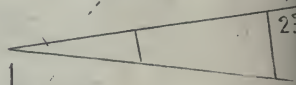
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25



26



23

Examples

Surveying

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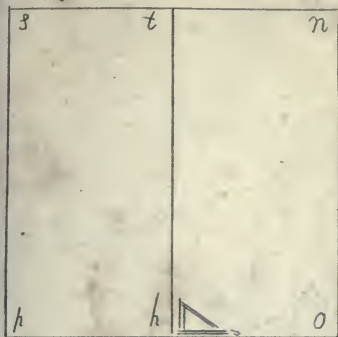


Fig 2

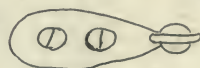


Fig 3

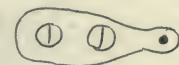
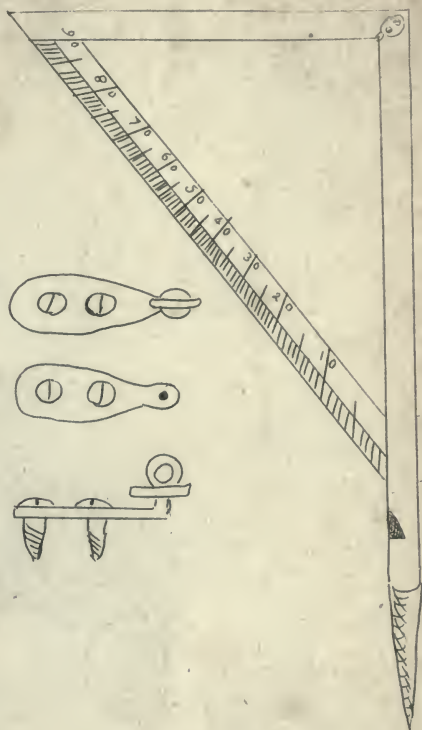
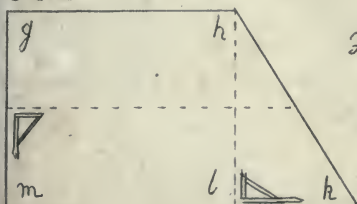


Fig 4



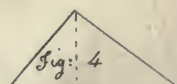
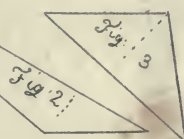
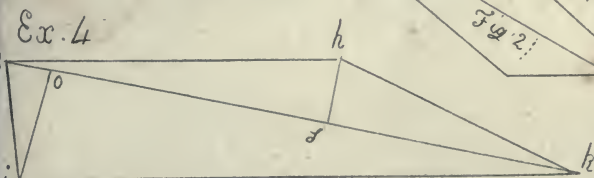
Ex. 2



Ex. 3



Ex. 4



Ex. 6

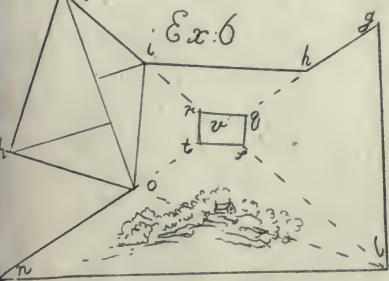
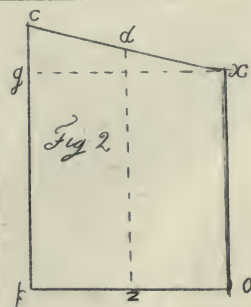
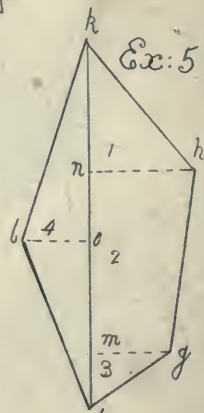


Fig 2

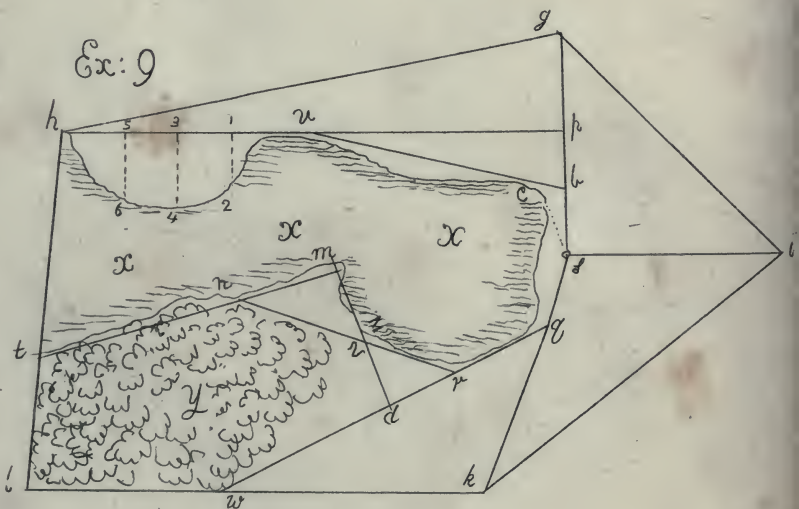
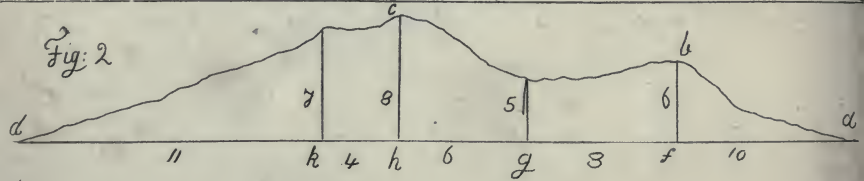
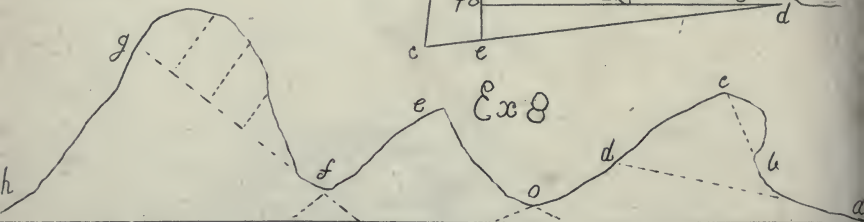
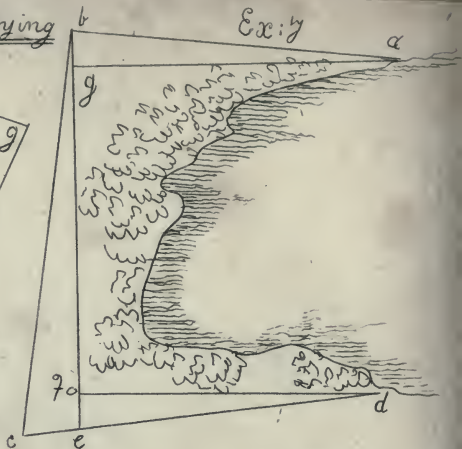
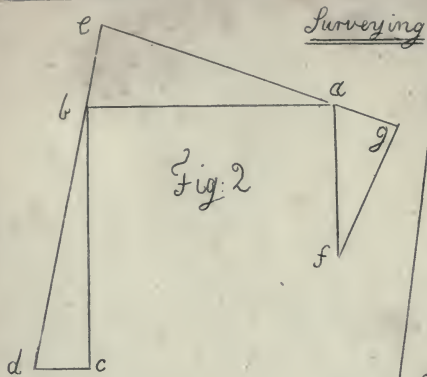


Ex. 5









Surveying

Fig: 1

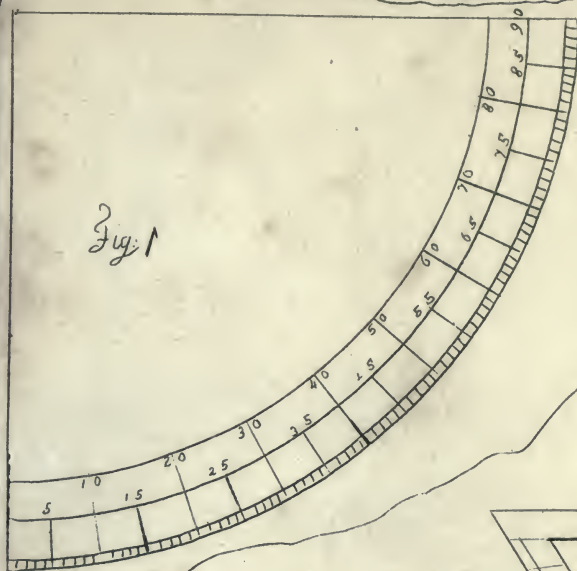


Fig: 2

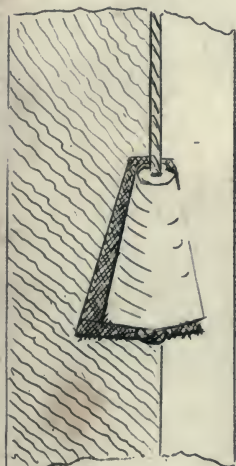
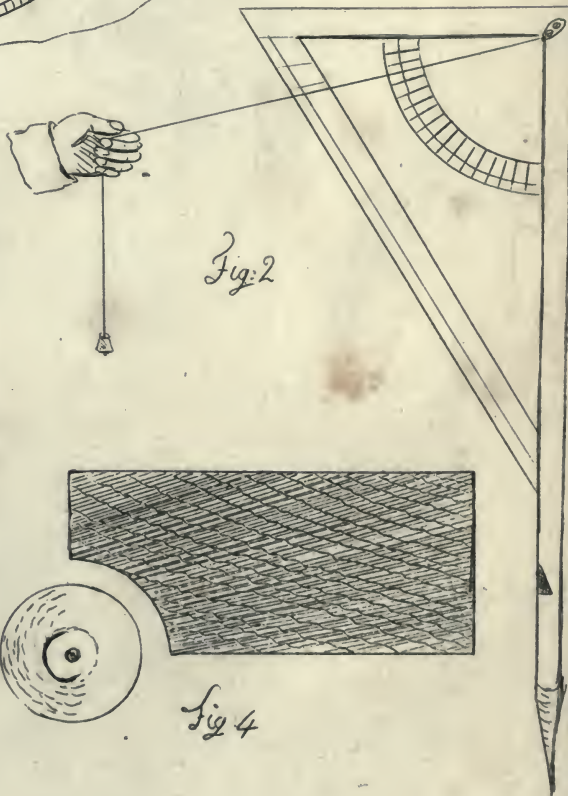


Fig: 3

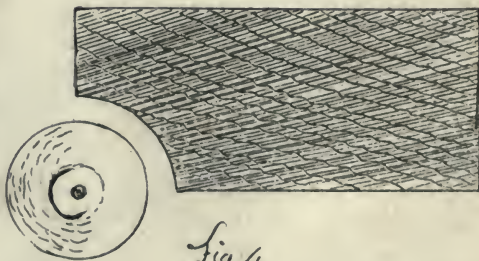


Fig: 4

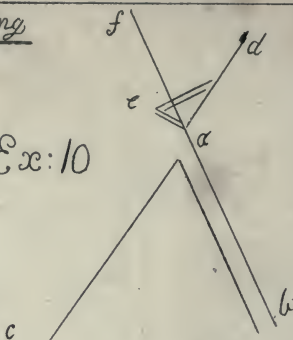




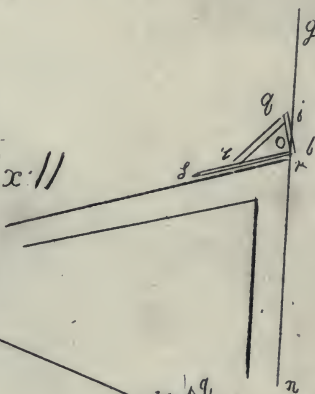
Surveying



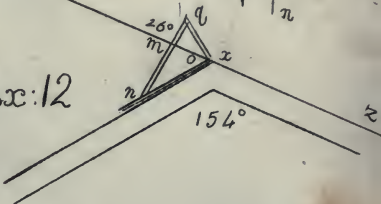
Ex: 10



Ex: 11



Ex: 12



Ex: 13



Ex: 14

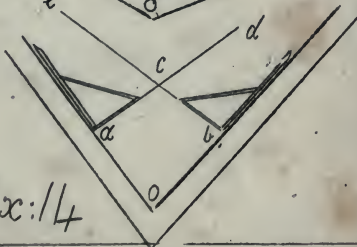


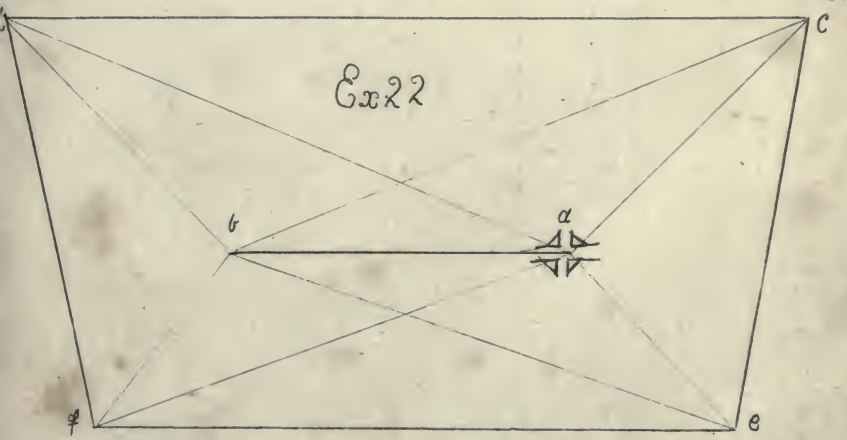
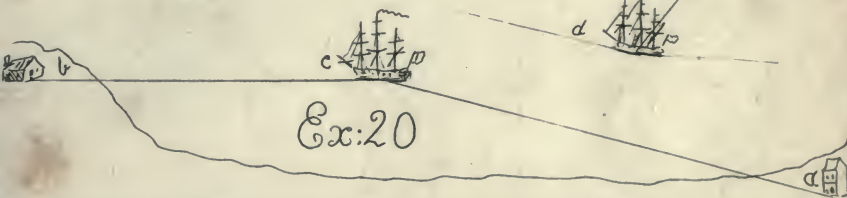
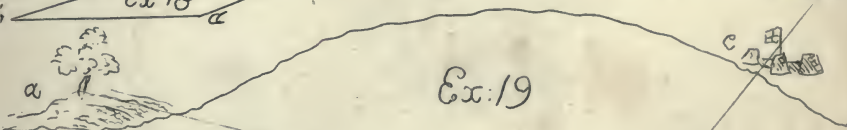
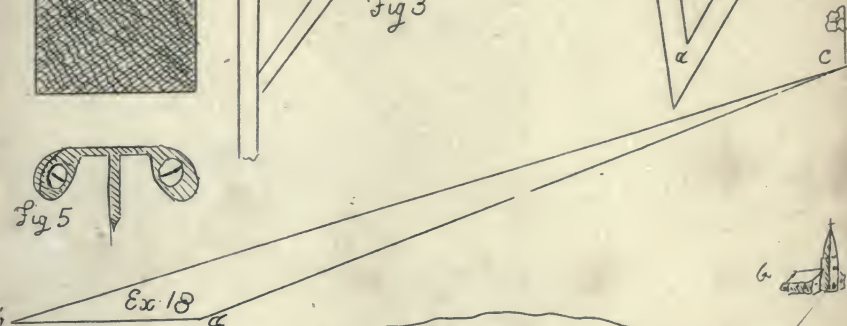
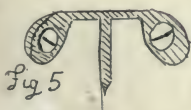
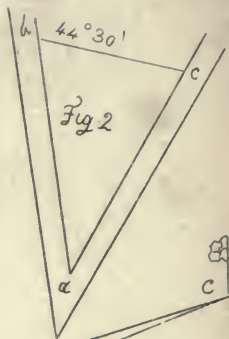
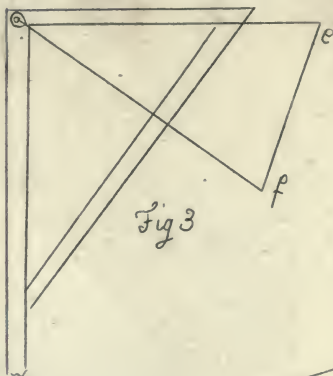
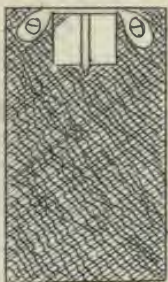
Fig: 5

Fig: 6



Surveying

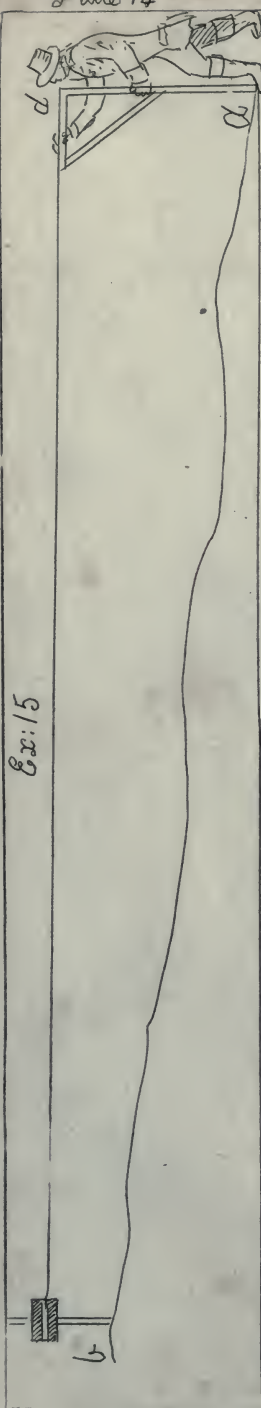
Fig:4





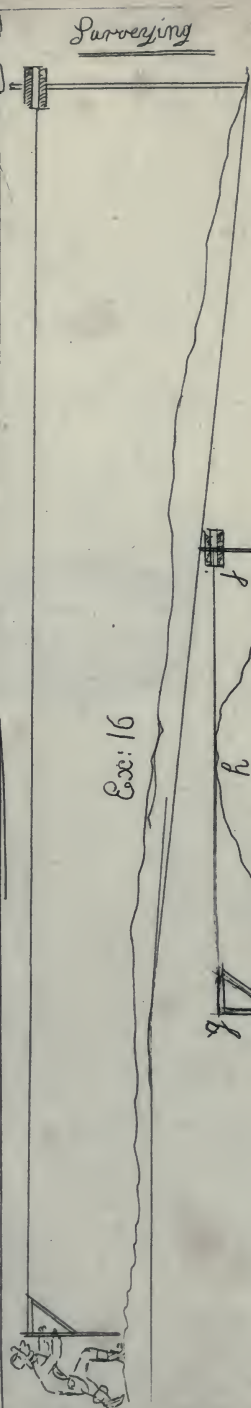


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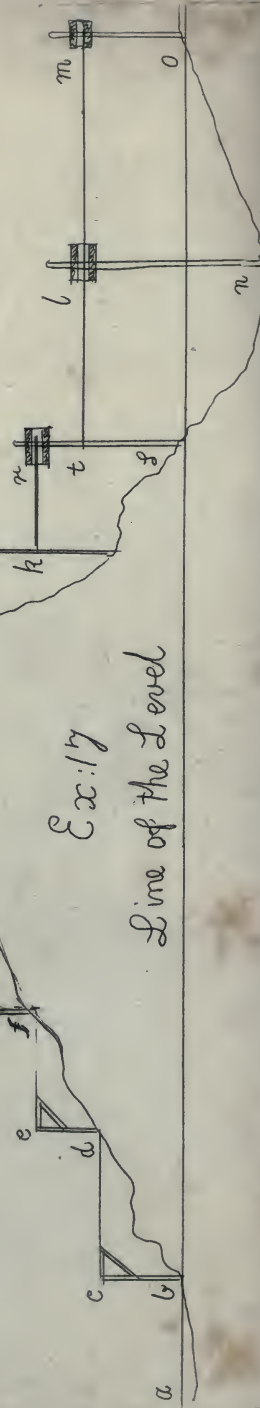
Surveying

Ex: 16



Ex: 17

Line of the Level



Ex: 21



Surveying

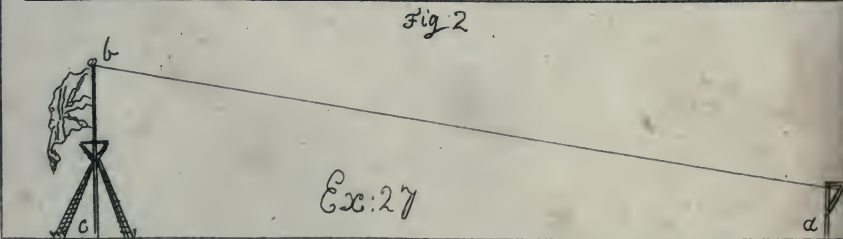
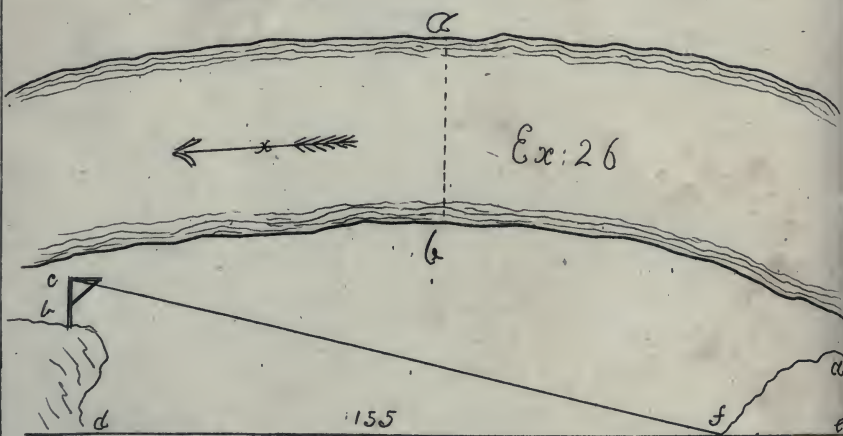
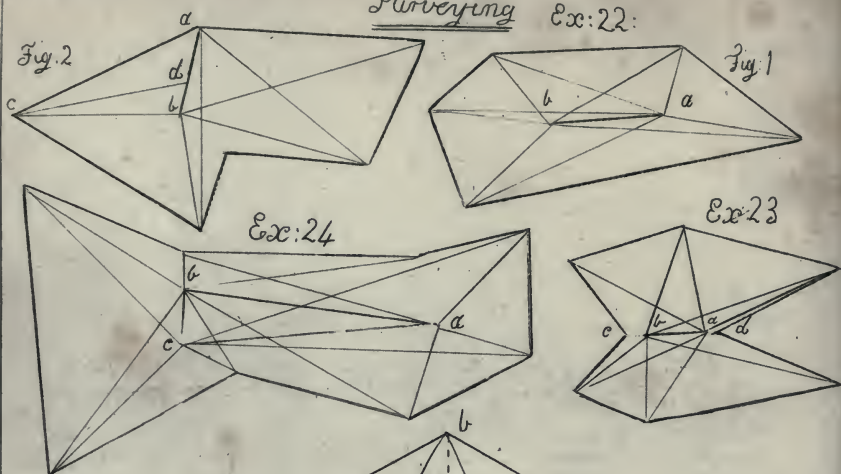
Fig. 1

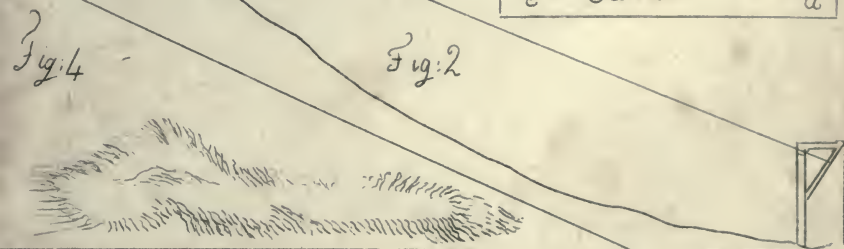
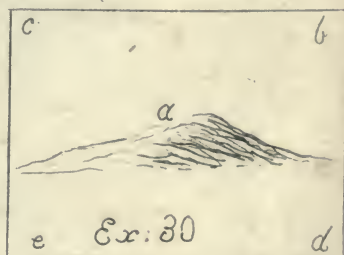
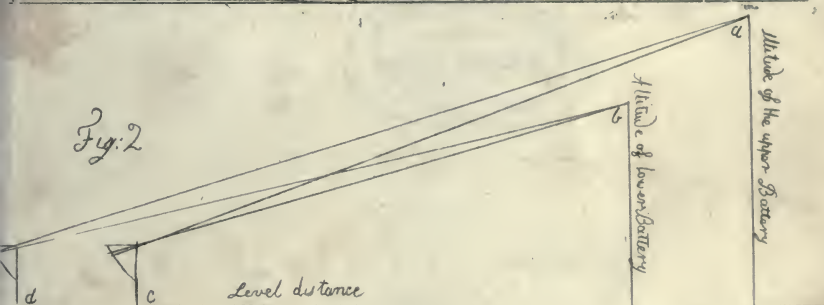
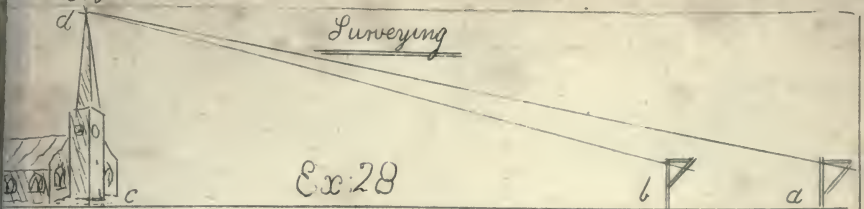
Fig. 2





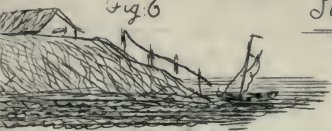
Surveying Ex:22:





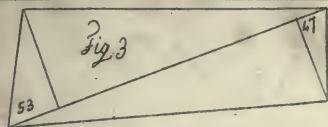






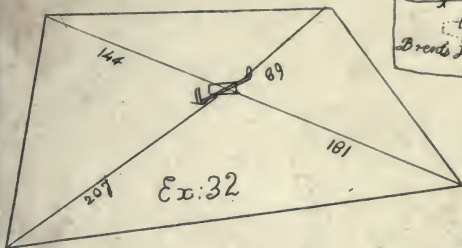
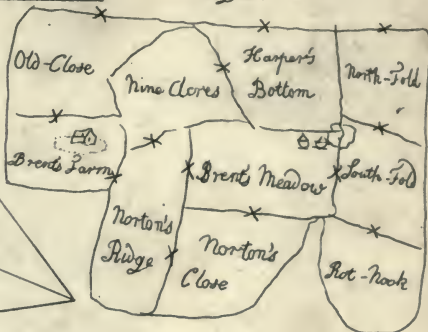
Ex: 31





Surveying

Fig 2



Ex 33

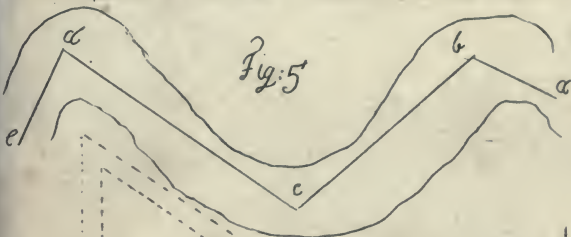
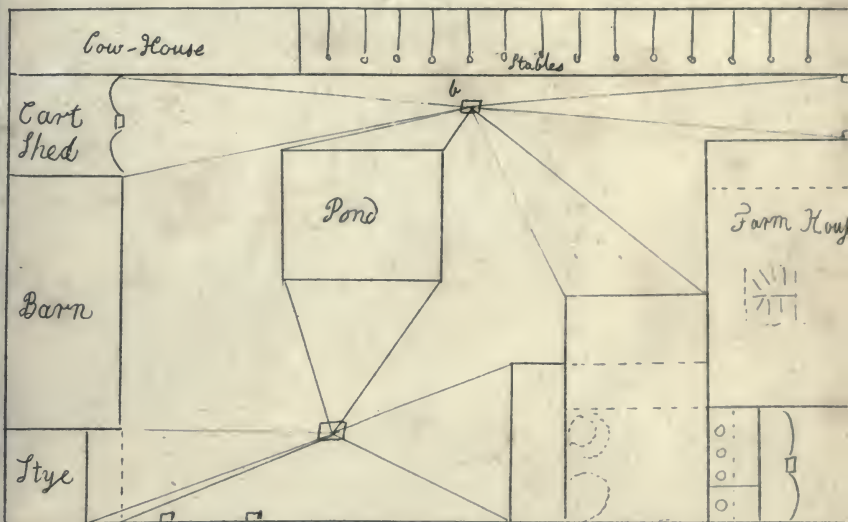


Fig 2

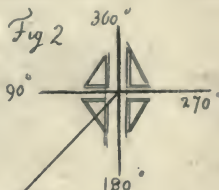
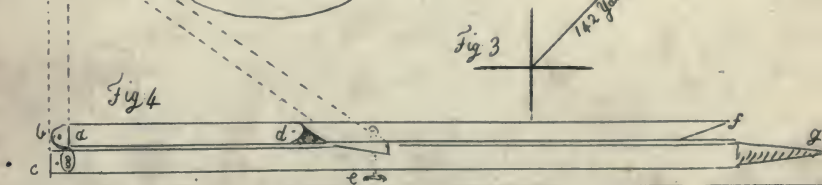


Fig 3







Surveying

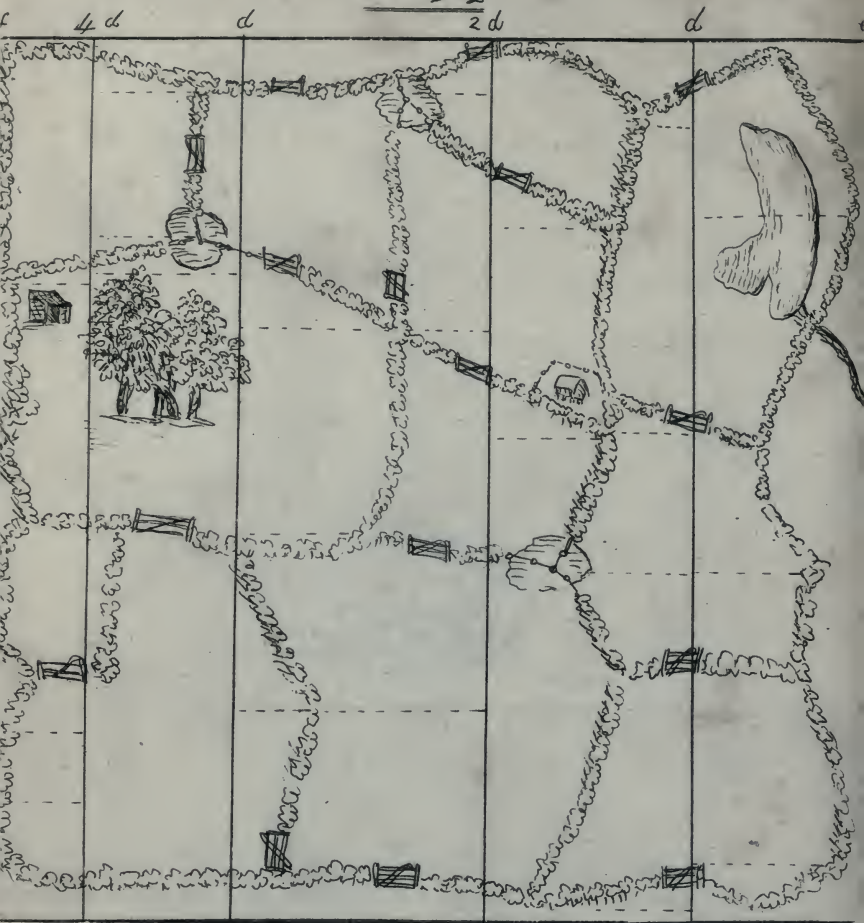


Fig. 3

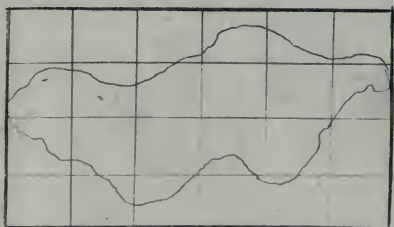


Fig. 2

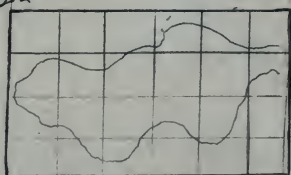
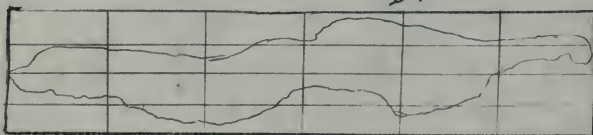


Fig. 4



Irrigation

Fig. 1

Simple Irrigation

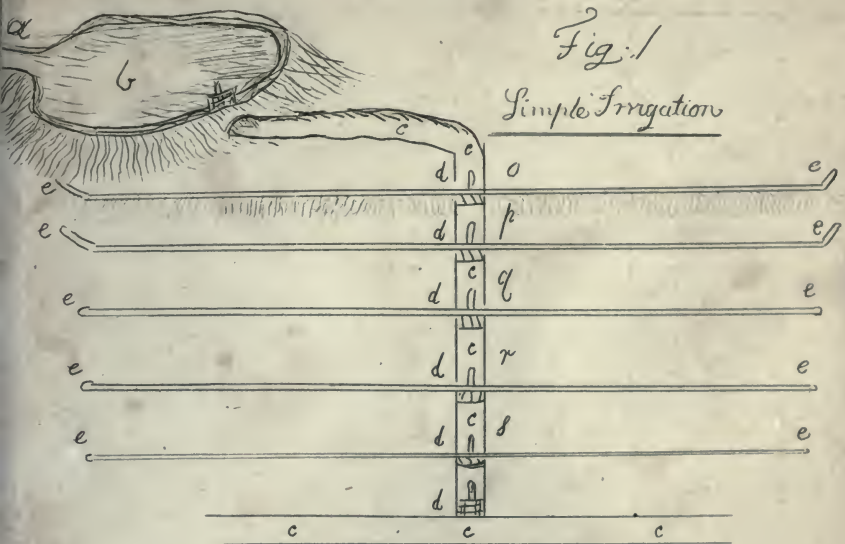
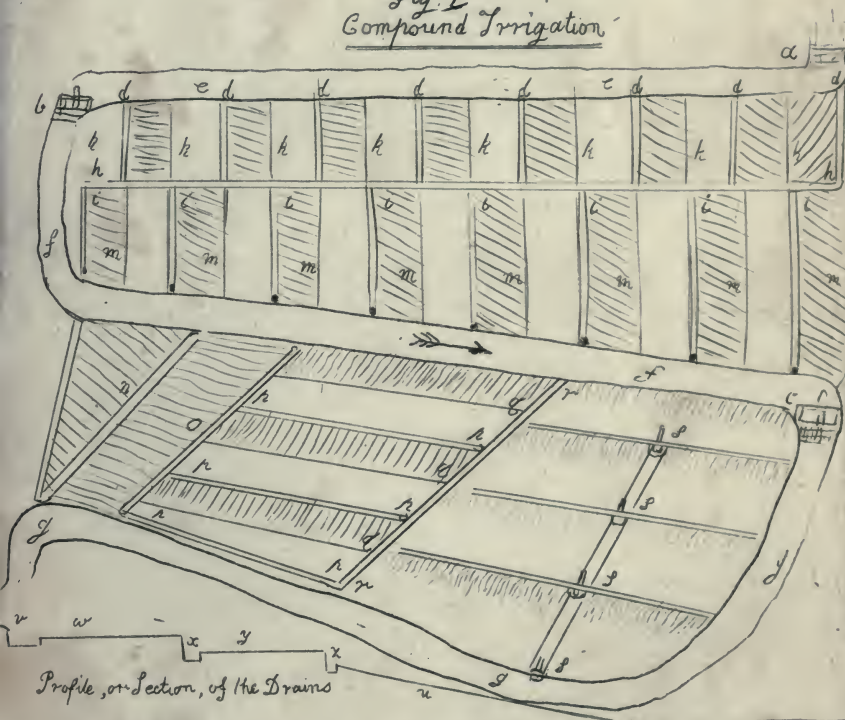


Fig. 2
Compound Irrigation







Timbers

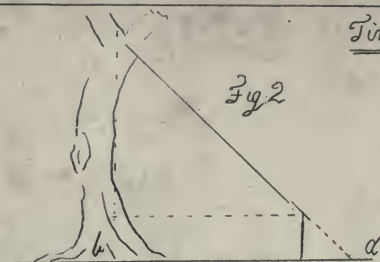


Fig. 1

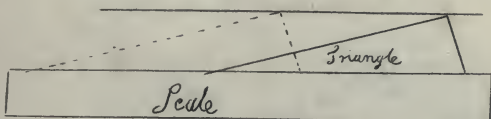
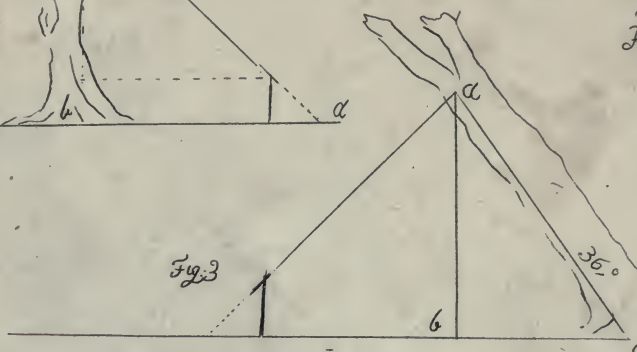
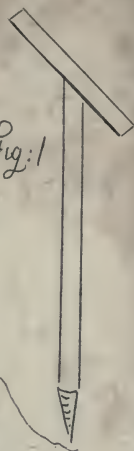


Fig. 5

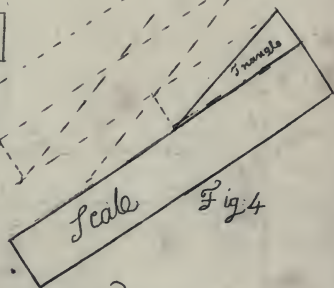
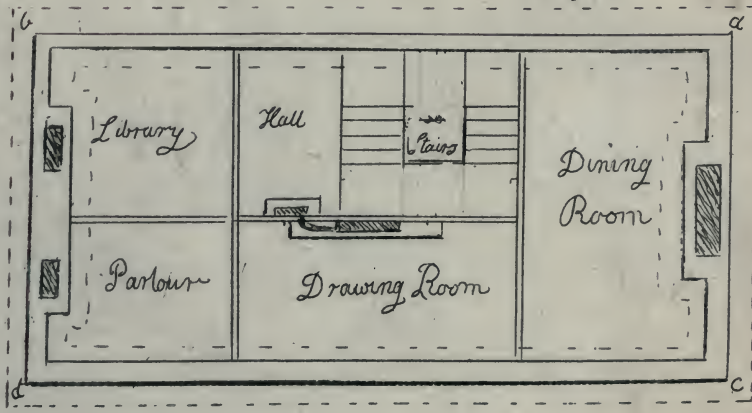
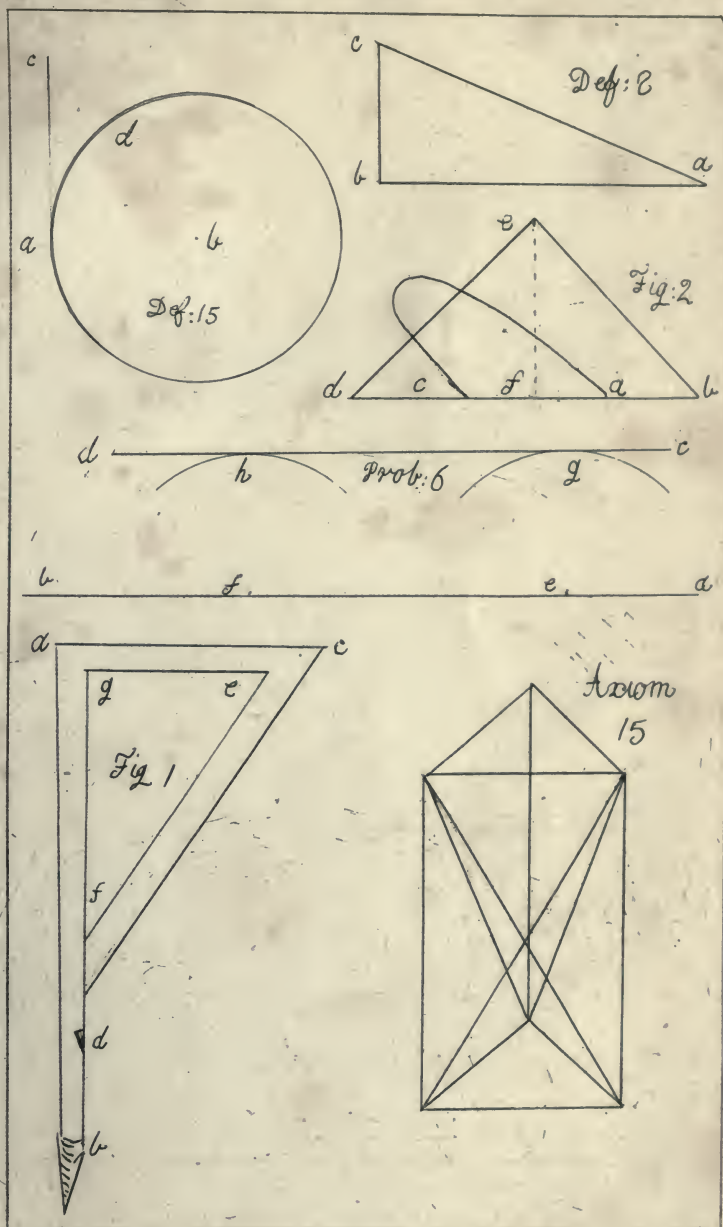


Fig. 4

Fig. 6



Supplementary Plate





5

